PBEE Assessment Methods

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Outline

1. **Conditional Probabilistic Approach**
   - ❑ **Introduction**
   - ❑ **SAC/FEMA**
   - ❑ **PEER PBEE (very brief)**
2. **Unconditional Probabilistic Approach**

Conditional Probabilistic Approach: Introduction

- Aimed to be practice-oriented
  - Currently employed mostly in the academic community
  - Expected to gain increasing acceptance in professional practice in the very near future
- Common standpoint of the methods: Use of intensity measure (IM) as an interface between seismology and structural engineering
  - IM is commonly represented with a hazard curve
  - *Structural engineers* need to have basic information about the process of obtaining a hazard curve, otherwise high potential to end up with incorrect seismic hazard representation

Excellent Review Article: Why Do Modern Probabilistic Seismic-Hazard Analyses Often Lead to Increased Hazard Estimates? By J.J. Bommer and N.A. Abrahamson [*Bulletin of the Seismological Society of America*, **96**(6):1967–1977, Dec. 2006]

Conditional Probabilistic Approach: SAC[(1)] / FEMA[(2)]

- During 1994 Northridge earthquake, some steel-moment-resisting-frame (SMRF) buildings underperformed by showing fractures in many beam-column joints which were supposed to remain elastic

- Originally developed for investigation of this unexpected behavior and assessing the seismic performance of these SMRF buildings

- Applicable to all building types with some adjustments

(1)SAC is a joint venture of the **S**tructural Engineers Association of California (SEAOC), the **A**pplied Technology Council (ATC), and **C**alifornia Universities for Research in Earthquake Engineering (CUREe), formed to address both immediate and long-term needs related to solving the problem of the WSMF connection.

(2)US **F**ederal **E**mergency **M**anagement **A**gency (FEMA) www.fema.gov

Conditional Probabilistic Approach: SAC / FEMA

- An empirical method based on assuming that an engineer uses ground motions and a computational model of a structure to test the likelihood that a building will perform as intended (**?**) over the period of interest

Ex. 1: Is the structure capable of remaining fully operational in a frequent earthquake ?

System Performance

Hazard Levels (Return Period)	Fully Operational	Operational	Life Safety	Near Collapse
Frequent (43 years)	●	o	o	o
Occasional (72 years)	Δ	●	o	o
Rare (475 years)	♦	Δ	●	o
Very rare (949 years)		♦	Δ	●

o: unacceptable performance

●: basic safety objective

Δ: essential hazardous objective

♦: safety critical objective

Ex. 2: Is the structure capable of surviving a very rare earthquake?

Conditional Probabilistic Approach: SAC/FEMA

- Can be considered as a special application of the more general PEER PBEE framework (***to be discussed later!***)
  - Complete consideration of uncertainty and probability
  - Performance assessment not with decision variables (DV)
  - Performance assessment considering
    - Intensity Measure (IM)
    - Engineering Demand Parameter (EDP)
    - Capacity of the Engineering Demand Parameter (ECP)
  - DV can be interpreted as a *binary* indicator of achieving the performance level:
    - **0**: unacceptable performance
    - **1**: acceptable performance

Conditional Probabilistic Approach: SAC/FEMA

Motivation for Consideration of Uncertainty

Traditional earthquake design (TED) philosophy:

- Prevent damage in low-intensity EQ (**50% in 50 years**)
- Limit damage to repairable levels in medium-intensity EQ (**10% in 50 years**)
- Prevent collapse in high-intensity EQ (**2% in 50 years**)

- If an engineer would accept that the world is deterministic, then in the case that he/she observes a structure not collapsing for the 2% in 50 years event, he/she could conclude that the probability of global collapse of the building would certainly be less than 2% in 50 years

- There are many sources of uncertainty in this problem that need to be taken into account for a realistic assessment of the collapse probability of this building

- These uncertainties will probably make the probability of global collapse much higher than 2% in 50 years

Conditional Probabilistic Approach: SAC/FEMA

Types and Sources of Uncertainty

Alea* (Latin)=*Dice

Aleatory uncertainty (*randomness*): The uncertainty inherent in a nondeterministic (stochastic, random) phenomenon.

Examples: The location and the magnitude of the next earthquake and the intensity of the ground shaking generated at a given site

Epist (Greek): Knowledge

Epistemic uncertainty: The uncertainty attributable to incomplete knowledge about a phenomenon that affects our ability to model it.

Examples: The definition of parameters and rules of a constitutive model for concrete

Conditional Probabilistic Approach: SAC/FEMA

Background

Total probability theorem:

Given n mutually exclusive events* $A_1,\ldots, A_n$ whose probabilities sum to 1.0, then the probability of an arbitrary event B:

$$p(B) = p(B|A_1)\,p(A_1) + p(B|A_2)\,p(A_2) + \ldots + p(B|A_n)\,p(A_n)$$

$$p(B) = \sum_i \left[p(B|A_i)\ p(A_i) \right]$$

$p(B|A_i)$: Conditional probability of B given the presence of A_i

$p(A_i)$: Probability of A_i

*Occurrence of any one of them automatically implies the non-occurrence of the remaining n−1 events

Conditional Probabilistic Approach: SAC / FEMA

Background

- Calculate the probability of exceedance (POE), **P**, of the i^th value of EDP

$$P\left(EDP^{i}\right)=\sum_{m} P\left(EDP^{i} \mid IM_{m}\right) p\left(IM_{m}\right)$$

Computational model & simulations

Hazard curve

- Calculate the probability (**p**) of EDP^i

$$\text{for } i = 1 : \# \text{ of EDP values}$$

$$p(EDP^{i}) = P(EDP^{i}) \quad \text{if } i = \# \text{ of EDP values}$$

$$p(EDP^{i}) = P(EDP^{i}) - P(EDP^{i+1}) \quad \text{otherwise}$$

Conditional Probabilistic Approach: SAC/FEMA

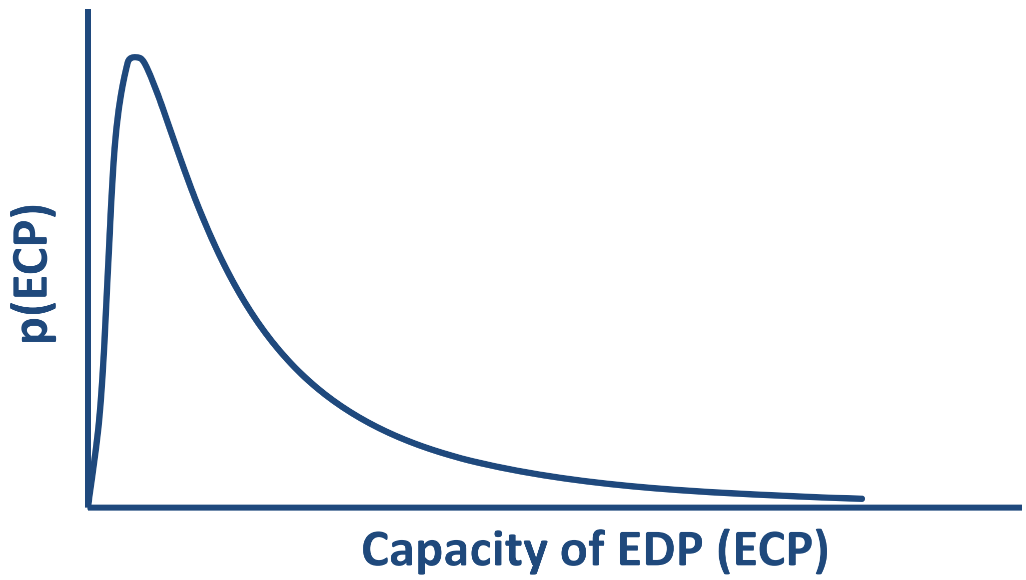
Background

- If an engineer was sure that the structure would fail its performance level when it reached a certain *limiting* EDP value (EDP^L), then the probability of not meeting that performance level (p_{fPL}) would be $P(EDP^L)$
- However, the engineer cannot be sure about the above issue, since there is uncertainty in the corresponding capacity limit
- Theoretically, every value of EDP has a finite likelihood of making a structure to fail a performance level
- Uncertainty in the capacity of an EDP (ECP) should be considered for the calculation of p_{fPL}
- Considering the uncertainty in capacity: p_{fPL} is defined as the probability of ECP being smaller than EDP [$p(ECP<EDP)$]
- Same uncertainty is considered in a different format in Damage Analysis stage of PEER PBEE framework

Conditional Probabilistic Approach: SAC / FEMA

Background

Uncertainty in capacity: Capacity of EDP that corresponds to a Performance Level (PL) is represented with a probability distribution



Conditional Probabilistic Approach: SAC / FEMA

Background

Table 6-7 *Modeling Parameters and Numerical Acceptance Criteria for Nonlinear Procedures— Reinforced Concrete Beams*

Conditions			Modeling Parameters[3]			Acceptance Criteria[3]				
			PR Plastic Rotation Angle, radians		Residual Strength Ratio	Plastic Rotation Angle, radians				
						Performance Level				
							Component Type			
							Primary		Secondary	
			a	b	c	IO	LS	CP	LS	CP
i. Beams controlled by flexure[1]										
$\frac{\rho-\rho'}{\rho_{bal}}$	Trans. Reinf.[2]	$\frac{V}{b_w d\sqrt{f'_c}}$								
≤ 0.0	C	≤ 3	0.025	0.05	0.2	0.010	0.02	0.025	0.02	0.05
≤ 0.0	C	≥ 6	0.02	0.04	0.2	0.005	0.01	0.02	0.02	0.04
≥ 0.5	C	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.03
≥ 0.5	C	≥ 6	0.015	0.02	0.2	0.005	0.005	0.015	0.015	0.02
≤ 0.0	NC	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.03
≤ 0.0	NC	≥ 6	0.01	0.015	0.2	0.0015	0.005	0.01	0.01	0.015
≥ 0.5	NC	≤ 3	0.01	0.015	0.2	0.005	0.01	0.01	0.01	0.015
≥ 0.5	NC	≥ 6	0.005	0.01	0.2	0.0015	0.005	0.005	0.005	0.01

FEMA-356

- If PR< 0.01 radians → PL = IO
- If 0.01<PR<0.02 → PL = LS
- If 0.02<PR<0.025 → PL = CP

No uncertainty in capacity

- PL = IO → p_{fPL} = P(PR=0.01)
- PL = LS → p_{fPL} = P(PR=0.02)
- PL = CP → p_{fPL} = P(PR=0.025)

Uncertainty in capacity

- PL = IO → $p_{fPL} \neq$ P(PR=0.01)
- PL = LS → $p_{fPL} \neq$ P(PR=0.02)
- PL = CP → $p_{fPL} \neq$ P(PR=0.025)

Reminder: **p**: probability, **P**: probability of exceedance (**POE**)

Conditional Probabilistic Approach: SAC / FEMA

Background

- Calculate the probability of exceedance (POE), **P**, of the i^th value of EDP

$$P\left(EDP^i\right) = \sum_m P\left(EDP^i \middle| IM_m\right) p\left(IM_m\right)$$

Computational model & simulations

Hazard curve

- Calculate the probability (**p**) of EDP^i

for i = 1 : # of EDP values

$$p(EDP^i) = P(EDP^i) \quad \text{if } i = \#\text{ of EDP values}$$

$$p(EDP^i) = P(EDP^i) - P(EDP^{i+1}) \quad \text{otherwise}$$

From slide 10

- Calculate the probability of not meeting a performance level (p_{fPL})

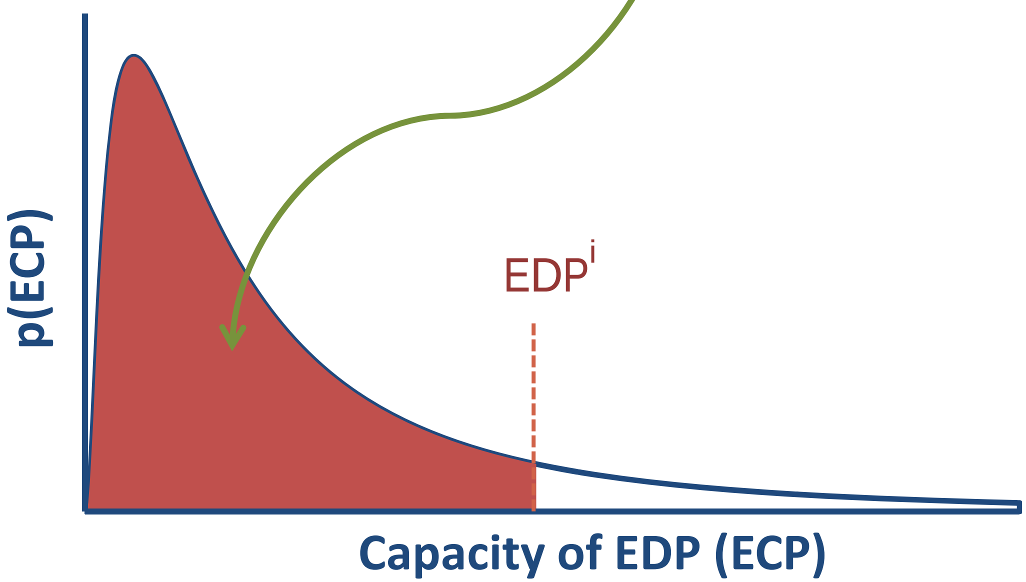
$$p_{fPL} = p[ECP \le EDP] = \sum_i p\left(ECP \le EDP^i \mid EDP^i\right) p\left(EDP^i\right)$$

Conditional Probabilistic Approach: SAC/FEMA

Background

- Calculate the probability of not meeting a performance level (p_{fPL})

$$p_{fPL} = p[ECP \leq EDP] = \sum_i p\left(ECP \leq EDP^i \mid EDP^i\right) p\left(EDP^i\right)$$



Conditional Probabilistic Approach: SAC/FEMA

Application Formats

- Approach requires large number of numerical simulations
- Computational effort introduced by the probability equations
- Two theoretically equivalent (with some practical differences) formats to reduce the computational burden:
  - Mean Annual Frequency (MAF) Format: A simple, closed-form evaluation of seismic risk (involving hazard, exposure, & vulnerability)
  - Demand and Capacity Factored Design (DCFD) Format: A check of whether the building satisfies the selected limit-state requirements

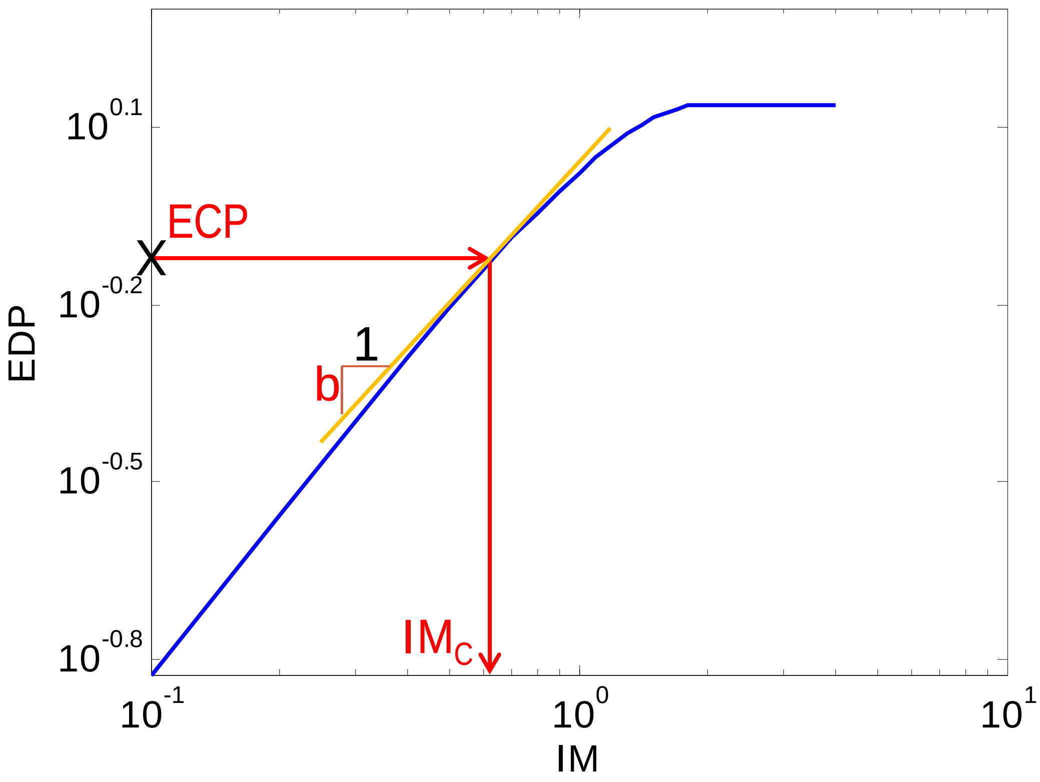
Conditional Probabilistic Approach: SAC / FEMA

Application Formats: MAF

Mean annual frequency of not meeting a certain performance level PL: λ_{fPL}

$$\lambda_{fPL} = H(IM_C)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DT}^2 + \beta_{CT}^2\right)\right]$$

IM_C: Value of IM that causes the structure to reach the EDP capacity (ECP) associated with the onset of the limit-state corresponding to the performance level PL.



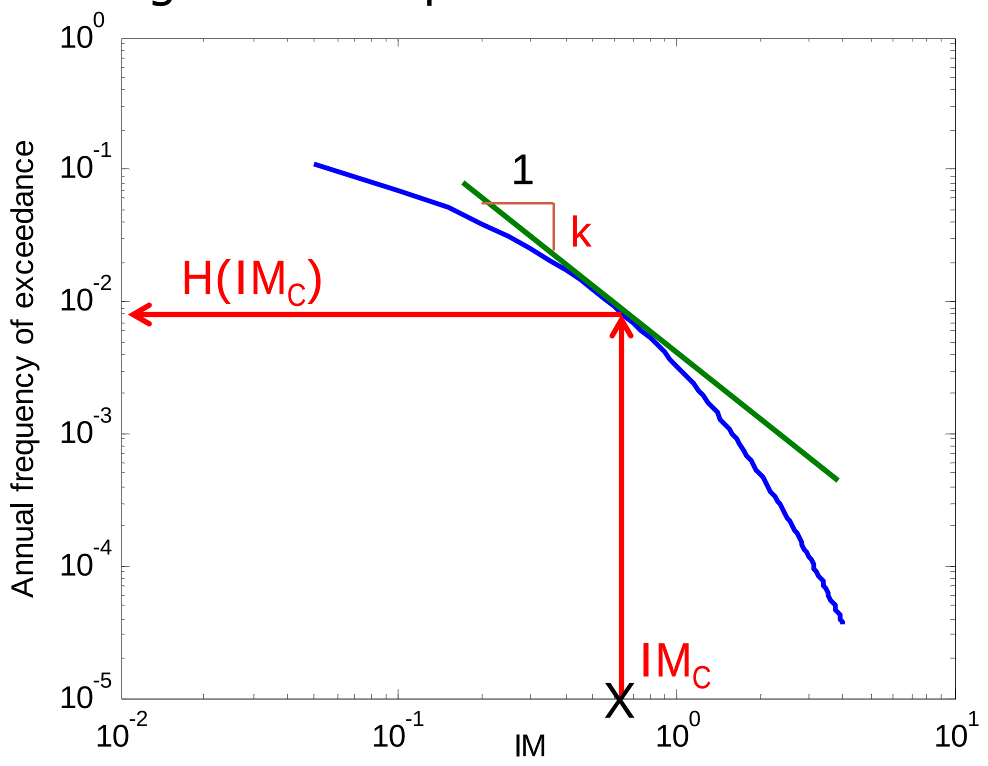
Conditional Probabilistic Approach: SAC / FEMA

Application Formats: MAF

Mean annual frequency of not meeting a certain performance level PL: λ_{fPL}

$$\lambda_{fPL} = H(IM_C)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DT}^2 + \beta_{CT}^2\right)\right]$$

$H(IM_C)$: Value of seismic hazard at IM_C



Conditional Probabilistic Approach: SAC/FEMA

Application Formats: MAF

Mean annual frequency of not meeting a certain performance level PL: λ_{fPL}

$$\lambda_{fPL} = H(IM_C)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DT}^2 + \beta_{CT}^2\right)\right]$$

Aleatory uncertainty (**R**andomness)

Epistemic **U**ncertainty

β_{DT}: Dispersion in **D**emand → $\beta_{DT} = \sqrt{\beta_{DR}^2 + \beta_{DU}^2}$

β_{CT}: Dispersion in **C**apacity → $\beta_{CT} = \sqrt{\beta_{CR}^2 + \beta_{CU}^2}$

Dispersion: Standard dev. of the log of the data

Aleatory Uncertainty

β_{DR}: Variability observed in structural response (Demand) from record-to-record
β_{CR}: Natural variability observed in tests to determine the EDP capacity (ECP) of a structural or non-structural component

Epistemic Uncertainty

β_{DU}: Uncertainty in modeling and analysis methods for estimating demand
β_{CU}: Incomplete knowledge of the structure for estimating capacity

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: MAF

Mean annual frequency of not meeting a certain performance level PL: λ_{fPL}

$$\lambda_{fPL} = H(IM_C)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DT}^2 + \beta_{CT}^2\right)\right]$$

Probability of not meeting a certain performance level PL: p_{fPL}

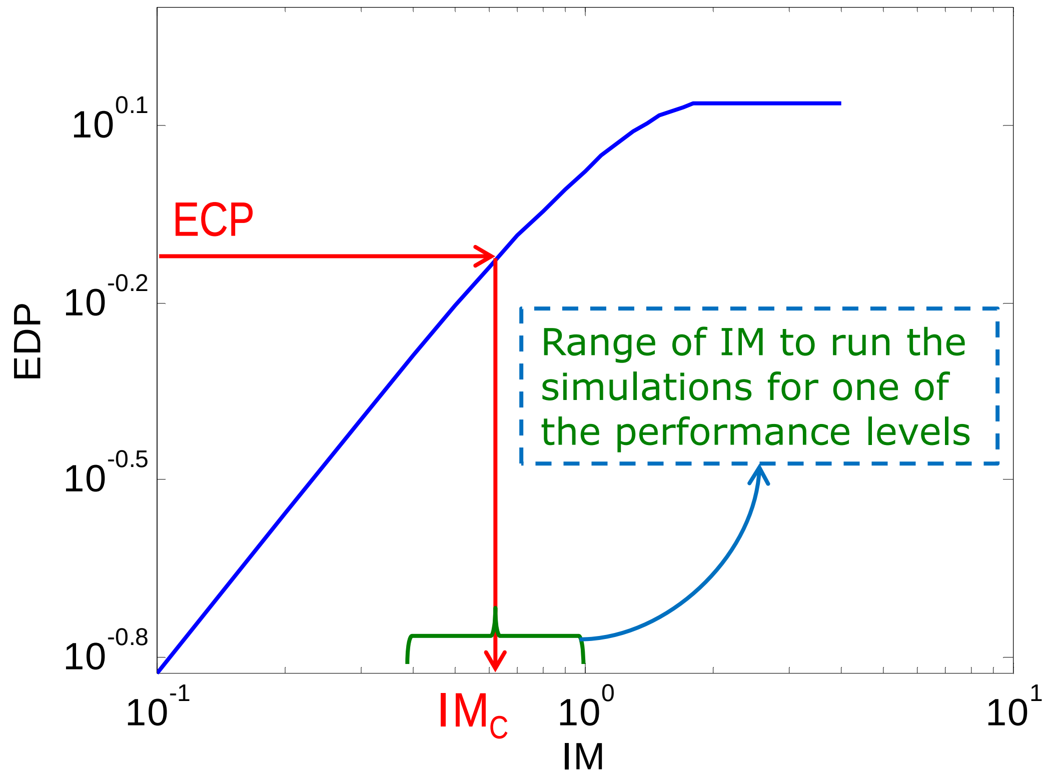
$$p_{fPL} = 1 - \exp\left(-\lambda_{fPL}\, t\right)$$ t: considered time period [years]

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: MAF

Advantage:

- Time history simulations do not need to be conducted for all IM values
- It may be sufficient to conduct the simulations for an estimated range of IM which covers the ECP values of the considered performance levels



Conditional Probabilistic Approach: SAC/FEMA

Application Formats: DCFD

- A check of whether a certain performance level has been met or violated
- Resembles the familiar Load and Resistance Factor Design (**LRFD**) of modern design codes
- Unlike the MAF format, it cannot provide an estimate of the annual frequency of exceeding a given performance level

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: DCFD

- FC: Factored capacity corresponding to the Performance Level
- FD_λ: Factored demand evaluated at the Hazard Level
- ECP_m: Median EDP capacity for the considered Performance Level
- $EDP_{m\lambda}$: Median demand evaluated at the IM level corresponding to λ

System Performance

Hazard Levels (Return Period)	Fully Operational	Operational	Life Safety	Near Collapse
Frequent (43 years)	●	o	o	o
Occasional (72 years)	Δ	●	o	o
Rare (475 years)	♦	Δ	●	o
Very rare (949 years)		♦	Δ	●

o: unacceptable performance
●: basic safety objective
Δ: essential hazardous objective
♦: safety critical objective

A performance objective:

Satisfy a Performance Level under a given Hazard Level

λ represents the annual frequency of exceedance associated with the Hazard Level

$$FC \geq FD_\lambda \Rightarrow \varphi \cdot ECP_m \geq \gamma \cdot EDP_{m\lambda},$$

$$\varphi = \text{Uncertainty Factor } (\sim \text{Strength Reduction Factor}),$$

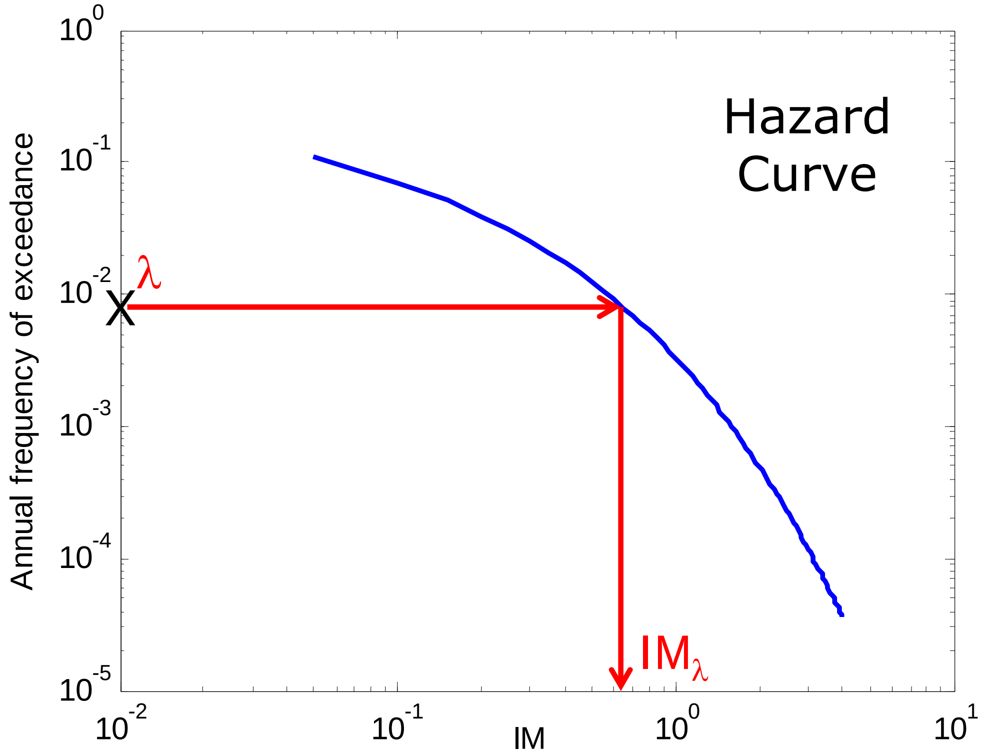
$$\gamma = \text{Uncertainty Factor } (\sim \text{Load Amplification Factor})$$

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: DCFD

$$FC \geq FD_{\lambda} \Rightarrow \varphi \cdot ECP_{m} \geq \gamma \cdot EDP_{m\lambda}$$

$EDP_{m\lambda}$: Median demand calculated at the IM value (IM_{λ}) corresponding to λ



Conditional Probabilistic Approach: SAC/FEMA

Application Formats: DCFD

$$FC \geq FD_{\lambda} \Rightarrow \varphi \cdot ECP_{m} \geq \gamma \cdot EDP_{m\lambda}$$

$$\varphi = \exp\left[-\frac{1}{2}\frac{k}{b}\left(\beta_{CR}^{2} + \beta_{CU}^{2}\right)\right] \qquad \gamma = \exp\left[\frac{1}{2}\frac{k}{b}\left(\beta_{DR}^{2} + \beta_{DU}^{2}\right)\right]$$

Remark:

- Median values are considered for capacity and demand
- Uncertainty is considered through the use of ϕ and γ
- Guarantees the Performance Objective with a confidence value greater than 50%
- Modifications have been made in DCFD to control and increase the confidence level: Enhanced DCFD (EDCFD)

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: EDCFD

$$FC_R \geq FD_{R\lambda} \cdot \exp(K_x \beta_{TU}) \Rightarrow \varphi_R \cdot ECP_m \geq \gamma_R \cdot EDP_{m\lambda} \cdot \exp(K_x \beta_{TU})$$

$$\varphi_R = \exp\left[-\frac{1}{2}\frac{k}{b}\beta_{CR}^2\right]$$

$$\gamma_R = \exp\left[\frac{1}{2}\frac{k}{b}\beta_{DR}^2\right]$$

Only Aleatory uncertainty

$$\beta_{TU} = \sqrt{\beta_{DU}^2 + \beta_{CU}^2}$$

Epistemic uncertainty

K_x: Standard normal variate (set of all random variables that obey a given probabilistic law) corresponding to the desired confidence level, α: $K_x = 1.28 \rightarrow \alpha=90\%$; $K_x = 0.00 \rightarrow \alpha=50\%$

EDCFD allows a user-defined **level of confidence** to be incorporated in the assessment.

Differing levels of confidence for:

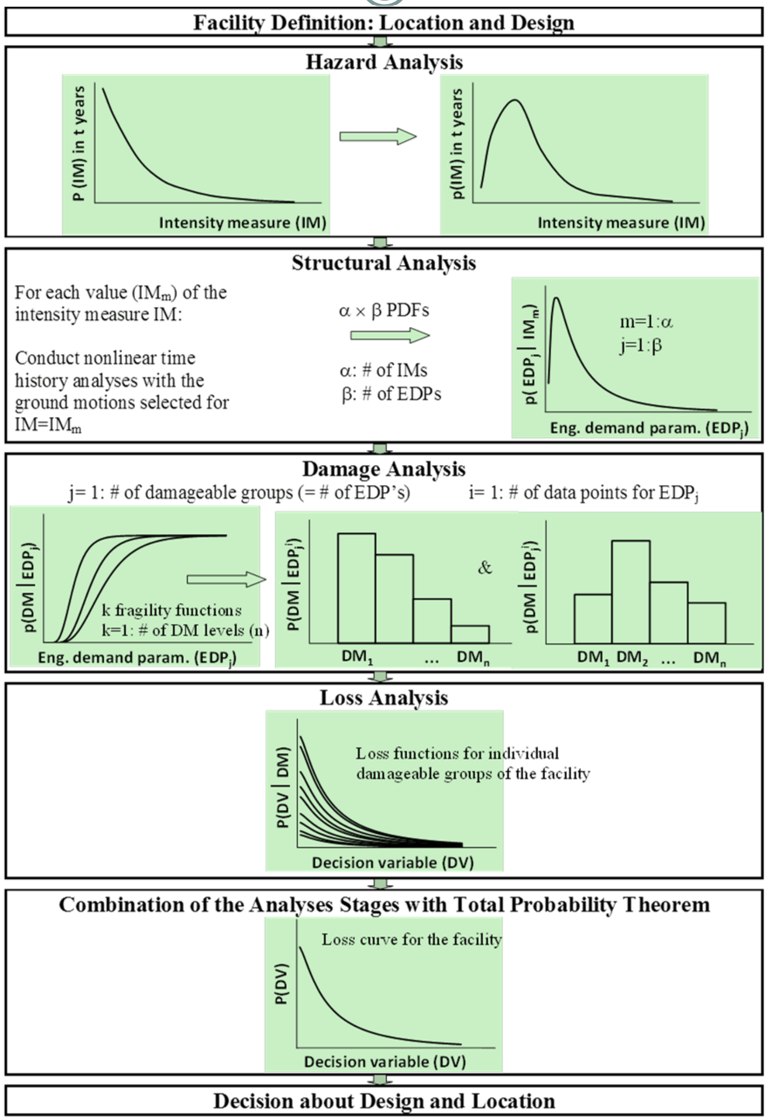
- Ductile versus brittle modes of failure (larger K_x for brittle)
- Local versus global collapse mechanisms (larger K_x for global)

Conditional Probabilistic Approach: PEER PBEE

- SAC/FEMA
  - Complete consideration of uncertainty and probability
  - Performance assessment not with decision variables (DV)
  - A special application of PEER PBEE framework

- PEER PBEE framework
  - Complete consideration of uncertainty and probability
  - Performance assessment with decision variables in terms of the direct interest of various stakeholders
  - Performance assessment considering:
    - Intensity Measure (IM)
    - Engineering Demand Parameter (EDP)
    - Damage Measure (DM)
    - Decision Variable (DV)

Conditional Probabilistic Approach: PEER PBEE



More about PEER PBEE in the afternoon

Unconditional Probabilistic Approach: Introduction

Conditional Probabilistic Approach (CPA)

- Practice-oriented
- Conditioned on IM
- Obtain the p(IM) from hazard curve
- Employ recorded ground motions compatible with IM

Replaced by

Unconditional Probabilistic Approach (UPA)

- More advanced
- Not conditioned on IM
- Stochastic models to directly describe the random time-series of seismic motion in terms of macro-seismic parameters, e.g. magnitude, distance, ... etc.

- Synthetic ground motions are employed in UPA
- The main difference with the CPA is in the to description of seismic motion at the site (synthetic motions)
- UPA-related research is mostly conducted up to generation of ground motions

Unconditional Probabilistic Approach: Introduction

- Methods of Unconditional Probabilistic Approach: Describe the randomness in the problem by a vector of random variables (**x**). **x** should ideally cover the randomness in:
  - Earthquake source
  - Propagation path
  - Site geology/geotechnical aspects
  - Frequency content of the time-series
  - Structural response and capacity

- Simulations for **x** sampled from its probability distribution, f(**x**)

Unconditional Probabilistic Approach

Simulation Methods

Simulation:

- A robust way to explore the behavior of systems of any complexity
- Based on the observation of system response to input

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \ldots & x_n \end{bmatrix}^T \quad \Rightarrow \quad f(\mathbf{x}): \text{probability distribution for } \mathbf{x}$$

- Form a set of inputs of **x** from f(**x**) $\quad \mathbf{x}^{<i>} = \begin{bmatrix} x_{1i} & x_{2i} & \ldots & x_{ni} \end{bmatrix}^T$
- Obtain the corresponding outputs
- Determine the distribution of the output through statistical post-processing

Unconditional Probabilistic Approach

Simulation Methods: Monte Carlo Simulation (MCS)

- A chosen set of inputs for **x**: $\mathbf{x}^{<i>} = [x_{1i} \quad x_{2i} \quad \ldots \quad x_{ni}]^T$

- If $\mathbf{x}^{<i>}$ fails in meeting certain performance requirements, then the contribution of $\mathbf{x}^{<i>}$ to the probability of not meeting those performance requirements (p_f) = $f(\mathbf{x}^{<i>})d\mathbf{x}$

- Then $p_f = \int_F f(\mathbf{x})d\mathbf{x}$

  F covers all $\mathbf{x}^{<i>}$ that fail in meeting the performance requirements

$$p_f = \int_F f(\mathbf{x})d\mathbf{x} = \int \underbrace{I_f(\mathbf{x})}_{\text{Indicator function}} f(\mathbf{x})d\mathbf{x} = E[I_f(\mathbf{x})]$$

$$\text{Indicator function} = \begin{cases} 1 & \text{if } \mathbf{x} \text{ belongs to F} \\ 0 & \text{otherwise} \end{cases}$$

Unconditional Probabilistic Approach

Simulation Methods: Monte Carlo Simulation (MCS)

$$p_f = \int_F f(\mathbf{x})d\mathbf{x} = \int I_f(\mathbf{x})f(\mathbf{x})d\mathbf{x} = E[I_f(\mathbf{x})]$$

Number of failed simulations

Monte Carlo Simulation: $p_f = E[I_f(\mathbf{x})] \cong \frac{1}{N}\sum_{i=1}^{N} I_f(\mathbf{x}^{<i>}) = \frac{N_f}{N} = \hat{p}_f$

Number of total simulations

- Obtain samples of $\mathbf{x}^{<i>}$ from the distribution f($\mathbf{x}$)
- Evaluate the performance of the structure for each $\mathbf{x}^{<i>}$
- Determine N_f and $\hat{p}_f$

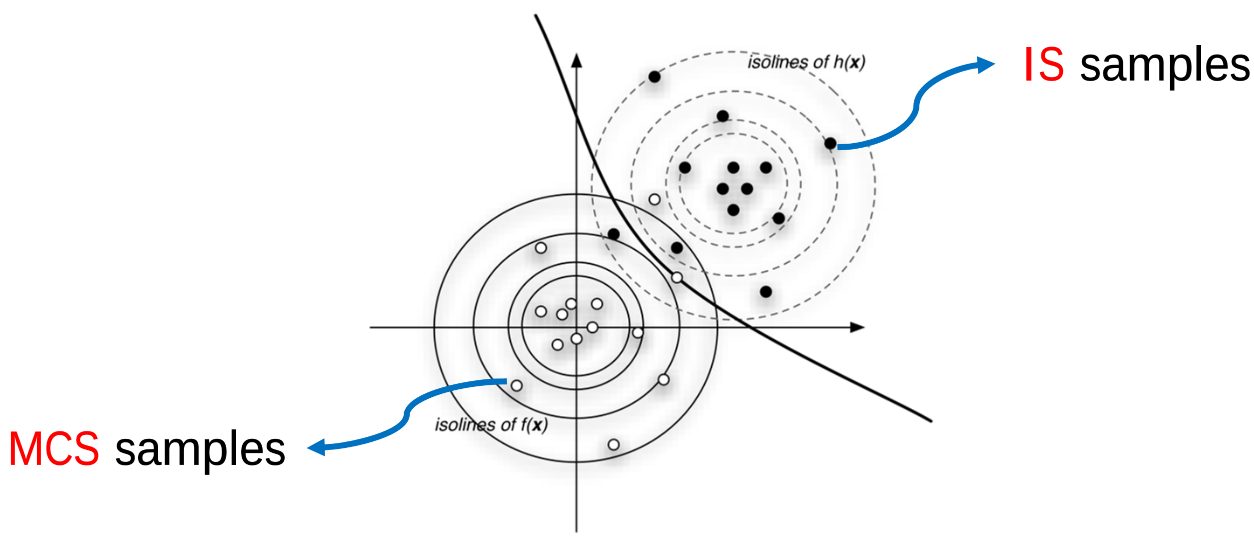
- $\hat{p}_f$ is an unbiased estimator of p_f
- Variance of $\hat{p}_f$ around p_f is proportional to p_f itself and decreases with increasing N

Unconditional Probabilistic Approach

Simulation Methods: Importance Sampling (IS)

- For very small values of p_f, N may need to be substantially large to obtain a few outcomes for N_f
- A possible solution to avoid excessive number of simulations $\longrightarrow$

Importance sampling (IS): Sample according to a more favorable distribution



Unconditional Probabilistic Approach

Simulation Methods: Importance Sampling (IS)

Importance Sampling: The different ways of sampling must be accounted for

$$p_f = \int_F f(\mathbf{x})d\mathbf{x} = \int I_f(\mathbf{x})\frac{f(\mathbf{x})}{h(\mathbf{x})}h(\mathbf{x})d\mathbf{x} = E_h\left[I_f(\mathbf{x})\frac{f(\mathbf{x})}{h(\mathbf{x})}\right] \cong \frac{1}{N}\sum_{i=1}^{N} I_f\left(\mathbf{x}^{<i>}\right)\frac{f(\mathbf{x}^{<i>})}{h(\mathbf{x}^{<i>})}$$

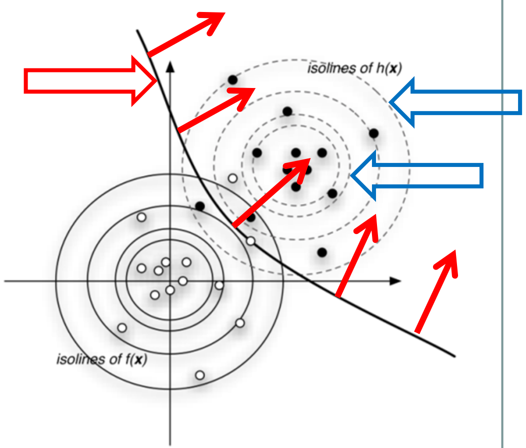
$$\frac{f(\mathbf{x})}{h(\mathbf{x})} = \alpha(\mathbf{x})$$

IS weight

Sampling density

Requires some knowledge of the failure domain F

Requires a good sampling density h(**x**)



Unconditional Probabilistic Approach

Simulation Methods: IS w/ K-means Clustering (IS-K) (Jayaram & Baker, 2010)

- For both MCS & IS methods, some of the samples could be redundant
- IS-K method identifies & combines redundant samples →
- Reduces the number of simulations further

In its simplest version, IS-K consists of **five** main steps:

Step 1: Pick (randomly) K samples

Step 2: Calculate the cluster centroids (typically mean of the K samples)

Step 3: Assign each sample to the cluster with the closest centroid

Step 4: Recalculate the centroid of each cluster after the assignments

Step 5: Repeat steps 1 to 3 until no more reassignments (in step 4) take place

Once all the events are clustered, a single random sample from each cluster is used to represent all samples in that cluster

Unconditional Probabilistic Approach

Methodology for Seismic Assessment

Vector of random variables **x**:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \mathrm{M} \\ \mathrm{Z} \\ \mathrm{D} \\ \mathrm{w} \\ \mathrm{SR} \\ \mathrm{STR} \end{bmatrix}$$

M →	Event magnitude M	Seismicity model
Z →	Active fault/zone	Seismicity model
D →	Source-to-site distance	Seismicity model
w →	Stationary white noise	Ground motion model
SR →	Soil	Site response model
STR →	Structure	Structural model

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Seismicity Model

Seismicity model parameters M, Z and D sampled using simulation

Sampling for M

Monte Carlo Simulation

$$f(m)=\frac{\sum_{i=1}^{n_f}\lambda_i f_i(m)}{\sum_{i=1}^{n_f}\lambda_i}$$

$f_i(m)$: probability distribution of M for the i^{th} fault/source

λ_i: activation frequency for the i^{th} fault/source

(mean annual rate of all events on the source, i.e. events with M>Lower bound M for that source)

n_f: # active faults/sources

Importance Sampling

$$h(m)=\frac{1}{n_m}\frac{f(m)}{\int_{m_k}^{m_{k+1}} f(m)dm}$$

$h(m)$: Sampling density for m lying in the k^{th} partition

n_m: # magnitude intervals (partitions) from m_{min} to m_{max}

Importance Sampling and K-means clustering

K-means clustering groups a set of observations into *K* clusters such that the dissimilarity between the observations within a cluster is minimized

Step 1: Pick (randomly) K samples

Step 2: Calculate the cluster centroids (typically mean of the K samples)

Step 3: Assign each sample to the cluster with the closest centroid

Step 4: Recalculate the centroid of each cluster after the assignments

Step 5: Repeat steps 1 to 3 until no more reassignments take place

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Seismicity Model

Seismicity model parameters M, Z and D sampled using simulation

Sampling for Z

Given that an earthquake with magnitude M = m has occurred, the probability that the event was generated in the i^{th} source is:

$$p(i|M=m)=\frac{\lambda_i f_i(m)}{\sum_{j=1}^{n_f}\lambda_j f_j(m)}$$

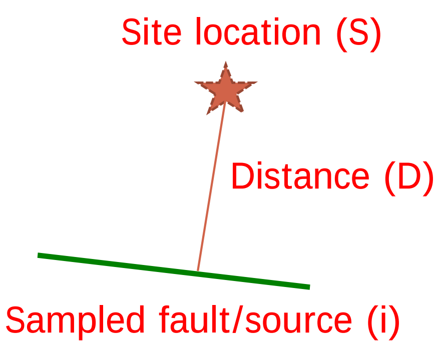
$f_i(m)$: probability distribution of M for the i^{th} fault/source
λ_i: activation frequency for the i^{th} fault/source
n_f: number of active faults/sources

Active zone (Z) is sampled from its discrete probability distribution conditioned on M

Sampling for D

There is no further effort needed to sample D.
It can be determined based on:

- The sampled fault/source
- The deterministic site location (S)



Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Synthetic Ground Motion Models:

- Seismologically-based Models
- Empirical Models

Seismologically-based Models

- Models that are based on the physical processes of earthquake generation and propagation
- Such models have reached a stage of maturity
- Applied in regions of the world where data is not sufficient for a statistical approach to seismic hazard
- Applied also in some regions of the world where seismic activity is well-known to (1) check their validity & (2) supplement existing information

Unconditional Probabilistic Approach

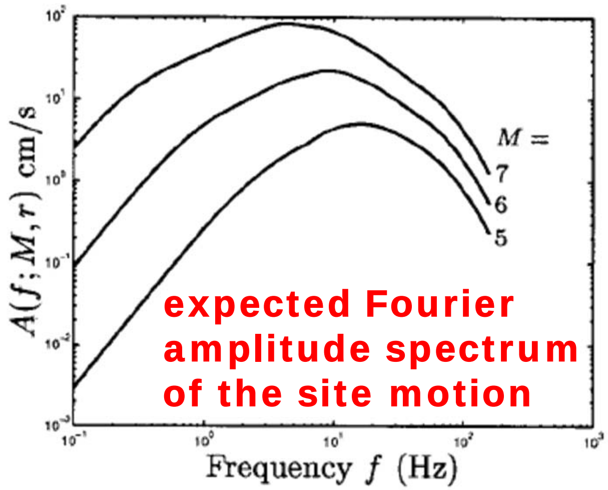
Methodology for Seismic Assessment: Ground Motion Model

Seismologically-based Models (Atkinson & Silva, 2000)

- Acceleration-amplitude Fourier spectrum (or Radiation spectrum)
- Generation of time history

Acceleration-amplitude Fourier spectrum (Au & Beck, 2003, Pinto et al, 2004)

$$A(\mathbf{f},\mathbf{M},\mathbf{R}) = A_0(f)\frac{1}{R'}\exp(-\gamma(f)R')\cdot\exp(-\pi f\kappa)V(f)$$ **&** $$A_0(f) = CM_0(2\pi f)^2\left[\frac{1-\varepsilon}{1+(f/f_a)^2}+\frac{\varepsilon}{1+(f/f_b)^2}\right]$$ **Source spectrum**



Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Acceleration-amplitude Fourier spectrum (Au & Beck, 2003, Pinto et al, 2004)

near-surface attenuation: κ = 0.03

$$A(\mathbf{f},\mathbf{M},\mathbf{R}) = A_0(f)\frac{1}{R'}\exp(-\gamma(f)R')\cdot\exp(-\pi f\kappa)V(f)$$

Anelastic attenuation

Geometric spreading factor for direct waves

$$A_0(f) = CM_0(2\pi f)^2\left[\frac{1-\varepsilon}{1+(f/f_a)^2}+\frac{\varepsilon}{1+(f/f_b)^2}\right]$$

Source spectrum

$f_a=10^{2.18-0.496M}$, $f_b=10^{2.41-0.408M}$ Corner frequencies

$M_0=10^{1.5(M+10.7)}$ Seismic moment

$C=C_R C_P C_{FS}/(4\pi\rho\beta^3)$

$C_R=0.55$ Average radiation pattern for shear waves

$C_P=2^{-0.5}$ Accounts for partition of waves in two horizontal components

$C_{FS}=2$ Free-surface amplification

ρ & β Density & shear-wave velocity in the vicinity of the source

$\varepsilon=10^{0.605-0.255M}$ Corner frequencies weighted through this parameter

$R'=\sqrt{h^2+R^2}$ Radial distance between source and site

R Epicentral distance

$h=10^{-0.05+0.15M}$ Nominal depth of fault [km] ranging from ~ 5 km for M=5 to 14 km for M=8

$\gamma(f)=\pi f/(Q\,\beta)$, $Q=180f^{\,0.45}$ Regional quality factor

$V(f)$ Describes the *amplification* through the crustal velocity gradient (wave passage)

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Seismologically-based Models

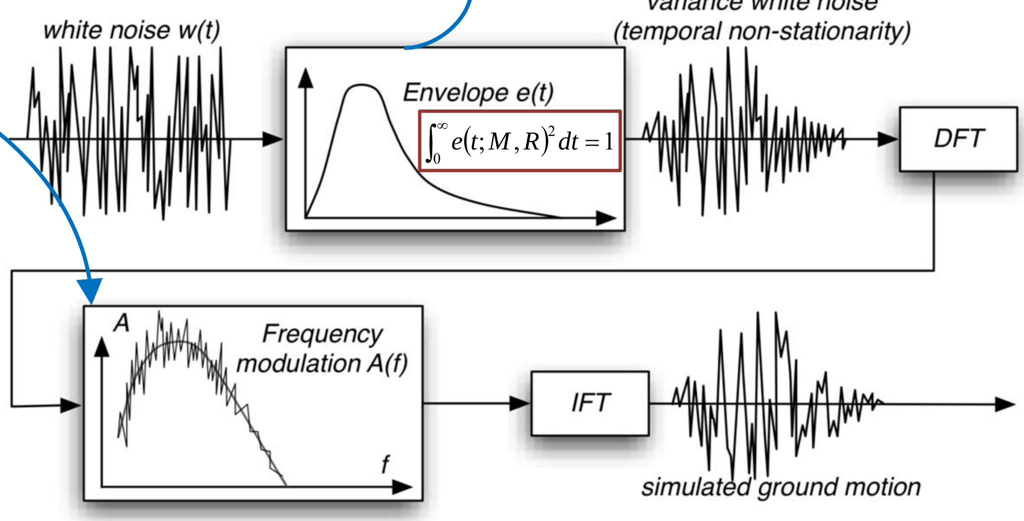
- Acceleration-amplitude Fourier spectrum (or Radiation spectrum)
- Generation of time history

Dependence on *M & R* introduced through α_3
α_1: Normalizing factor → envelope has **unit energy**
U(*t*): Unit-step function

Generation of time history

$$e(t;M,R)=\alpha_1 t^{\alpha_2-1}\exp(-\alpha_3 t)U(t)$$

multiply



DFT: Discrete Fourier Transform

IFT: Inverse Fourier Transform

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Synthetic Ground Motion Models:

- Seismologically-based Models
- Empirical Models

Empirical Models

- Models consist of parameterized stochastic (random) process models
- Developed by observing that ground motions possess stable statistical nature given earthquake and site characteristics (M, R, & soil type)
- This observation led to the idea of considering the ground motion acceleration time-series as samples of random processes

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Synthetic Ground Motion Models:

- Seismologically-based Models
- Empirical Models

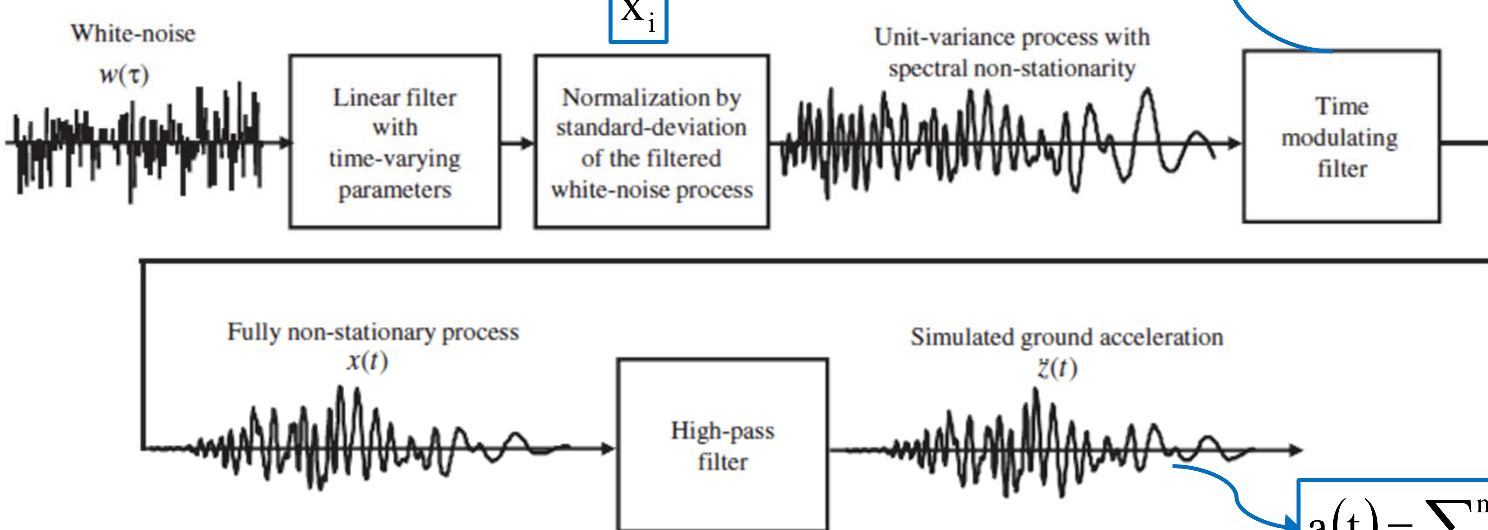
$\omega_g = \omega_g(t)$
$\xi_g \; \xi_g(t)$

Empirical Models (Rezaeian & Der Kiureghian, 2010)

The filter **IRF** (Impulse Response Function) is the acceleration IRF of a linear SDOF oscillator of natural frequency ω_g and damping ratio ξ_g

$$h_i(t) = h(t-\tau_i) = \frac{\omega_g}{\sqrt{1-\xi_g^2}} \exp\left[-\xi_g \omega_g (t-\tau_i)\right] \sin\left[\omega_g \sqrt{1-\xi_g^2}\,(t-\tau_i)\right]$$

x_i



$$a(t) = \sum_{i=1}^{n} x_i h_i(t)$$

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Site-Response Model

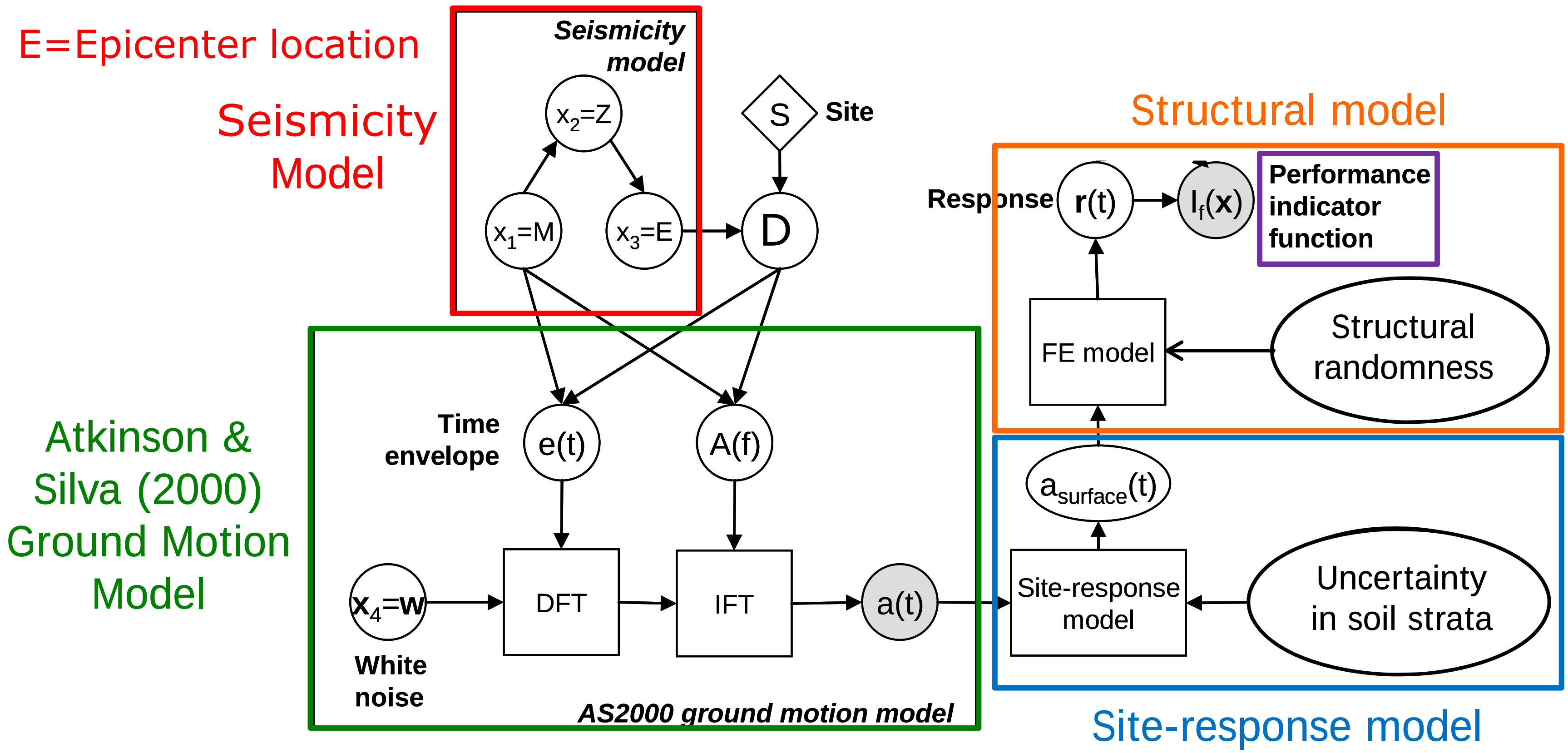
- Ground motion model determines ground motion time history for bedrock
- A site response model is used to obtain input motion to the structure at the surface
- Model the soil strata and corresponding stiffness, strength, & damping properties, e.g. a one-dimensional nonlinear, or equivalent linear, model)
- The strata thicknesses and properties possess uncertainty

Methodology for Seismic Assessment: Structural Model

- Finite element model which determines the response of the structure
- Both the structure itself, and the response-model implemented in the analysis software, are affected by uncertainty (more in the afternoon)

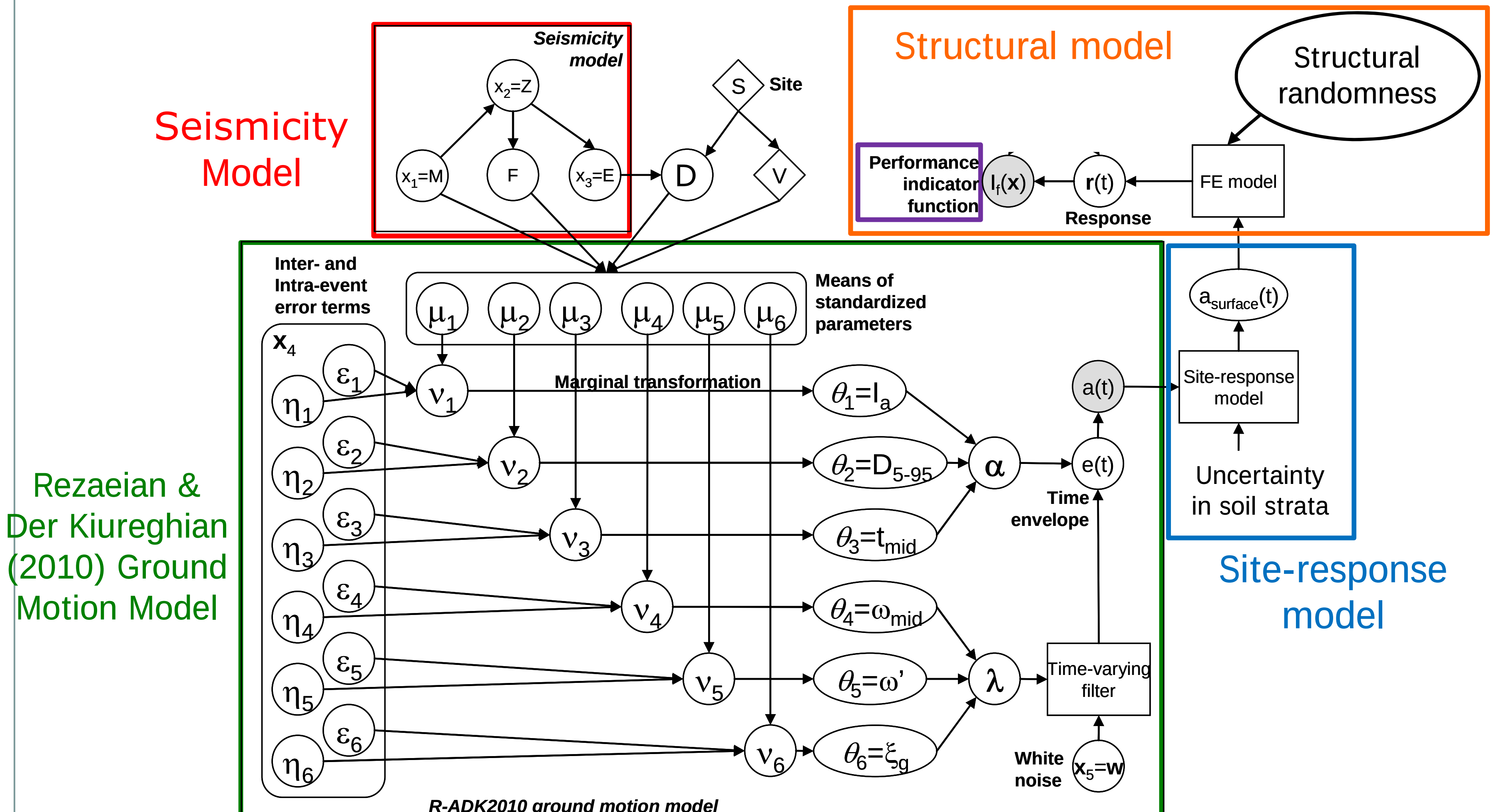
Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Flowchart



Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Flowchart



Unconditional Probabilistic Approach

Methodology for Seismic Assessment:

Application to the Estimation of a Structural Mean Annual Frequency

POE of PL given an event occurs from the i^{th} source

MAF of a performance level (PL)

$$\lambda_{PL} = \sum_{i=1}^{N} \lambda_i P_{PL|i} = \lambda_0 \sum_{i=1}^{N} \left(\lambda_i / \lambda_0\right) P_{PL|i} = \lambda_0 \sum_{i=1}^{N} P_{PL|i} \cdot p_i = \lambda_0 P_{PL}$$

$$\lambda_0 = \sum_{i=1}^{N} \lambda_i$$

i^{th} seismic source

POE of PL: could be calculated with the seismic assessment flowchart (**2 previous slides**)

λ_i: activation frequency for the i^{th} fault/source

(mean annual rate of all events on the source, i.e. events with M>Lower bound M for that source)

Thank you