PBEE Design Methods

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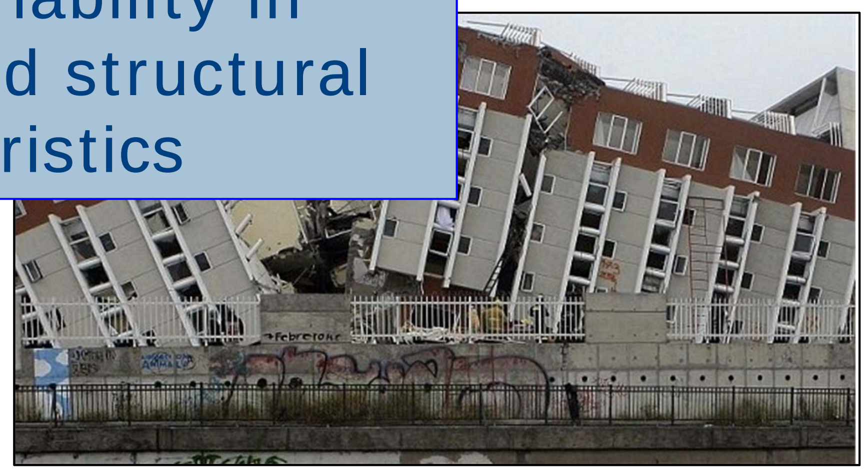
Outline

1. Introduction
2. Optimization-based methods
3. Non optimization-based methods

Introduction

Courtesy of Prof. S. Mahin

Robust structures and systems needed to account for great variability in earthquake and structural characteristics



Introduction

- Performance of a structure under earthquake excitation depends on:
  - Earthquake characteristics
  - Proximity to fault rupture
  - Soil and foundation type
  - Structural system
  - Configuration and details
  - Nonstructural components
  - Quality of engineering
  - Quality of construction
- Probabilistic seismic design is the direct design method which considers the uncertainty and variability of the above items
- The state of development of fully probabilistic seismic design methods is behind that of assessment methods

Optimization-based methods

Structural optimization problems can be expressed as:

Vector of decision variables

$$\min f(\mathbf{x}) \text{ subject to } g(\mathbf{x} \leq \alpha)$$

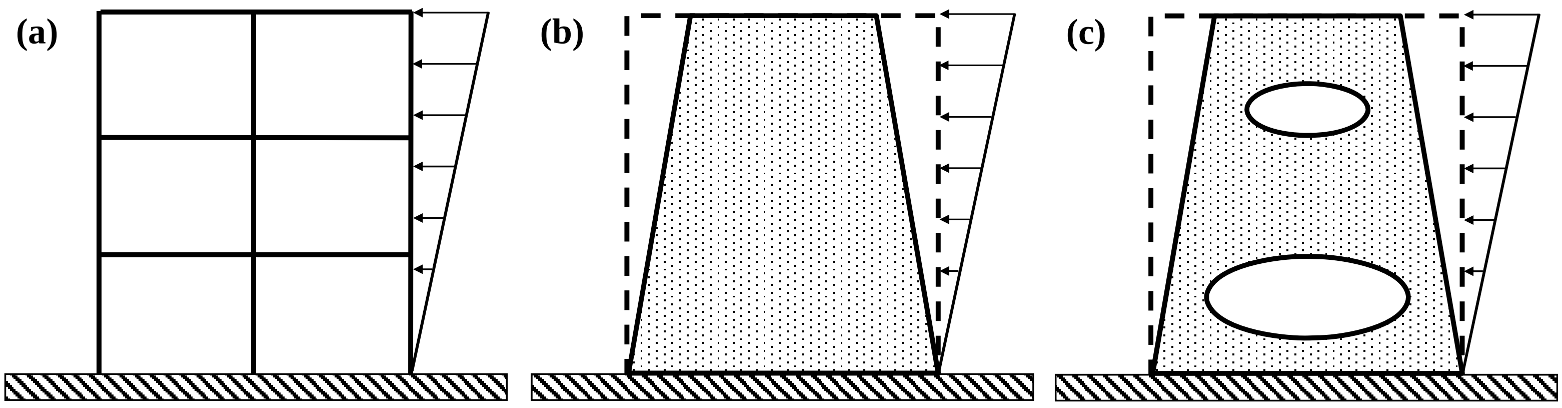
Objectives Constraints

Although this is a common notation for almost all optimization problems, the structure being optimized, variables, constraints and the domain of optimization can be significantly different.

Optimization-based methods

Classification

Sizing | Shape | Topology



Sizing

- Both the size and location of structural members are varied
- Dimensions are varied to obtain the optimal solutions

Shape

- Connectivity of the structure is fixed
- Shape (boundary) is varied to obtain the optimal solution

Topology

- Locations and number of elements are fixed and known
- Formation of new boundaries is allowed

Most optimization studies on structural earthquake engineering deal with sizing, where the design variables are limited to member/section properties

Optimization-based methods

Terminology

- Objective (merit) function: A function that measures the performance of a design
  - Takes a different value for every design alternative
  - **Ex.**: Maximum interstory drift ratio (MIDR), initial cost, …
- Design (decision) variables: A vector that defines the design
  - Each element in the vector describes a different structural property relevant to the optimization problem
  - Take different values throughout the optimization process
  - **Ex.**: Section dimensions, reinforcement ratios, …
- Constraint: A condition that a solution of the optimization problem should satisfy
  - **Ex.**: Traditional code design requirements

Optimization-based methods

Terminology

- Space of design (decision) variables (search space): Space defined by the range of design (decision) variables
  - k dimensions: k is the number of design variables in the problem
  - Each dimension: either continuous or discrete depending on the nature of the corresponding design variable

- Solution (objective function) space: Space defined by the objective function
  - Usually the solution space is unbounded or semi-bounded
  - n dimensions: n is the number of objective functions in the problem
  - The optimal solutions are defined in the solution space
  - The set of optimal solutions in the solution space is referred to as a Pareto-front or Pareto-optimal set

Vilfredo Pareto (1848–1923): Italian economist

Optimization-based methods

Terminology

- Pareto-optimality:

Minimize $f(\mathbf{x})$

the objective function

$\mathbf{y}^{<i>} = f(\mathbf{x}^{<i>})$

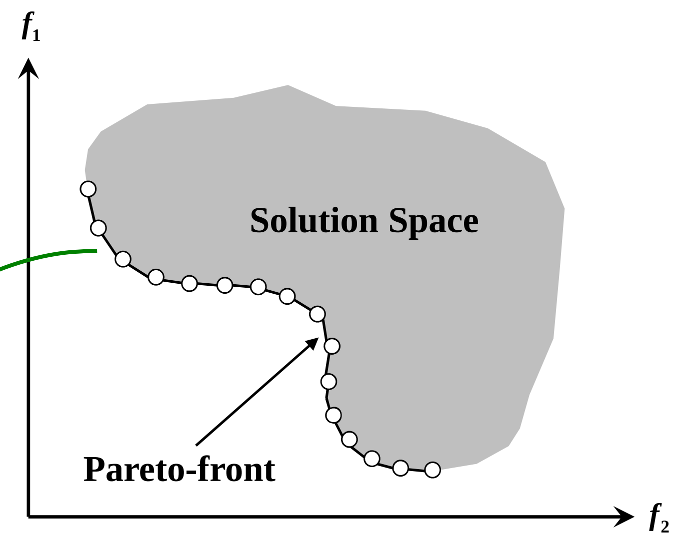
$\mathbf{x}^{<i>}$: A point in the search space

$\mathbf{y}^{<i>}$: Corresponding point in the solution space

$$\mathbf{y}^{<i>} \text{ dominates } \mathbf{y}^{<j>} \text{ if } \mathbf{y}_n^{<i>} < \mathbf{y}_n^{<j>} \text{ for all n}$$

$$\text{if } \mathbf{y}_n^{<i>} < \mathbf{y}_n^{<j>} \textit{ strictly} \text{ for at least a single n}$$

Pareto-front (Y*) is the subset of points that are not strictly dominated by another point.



Boundary that minimizes objectives f_1 and f_2

Optimization-based methods

Terminology

- Performance levels: Levels that describe the performance of the structure against earthquake hazard
  - Exceedance of each performance level is determined based on the crossing of a threshold value (with a probabilistic distribution) in terms of structural capacity
  - **Ex.**: Immediate Occupancy (IO), Life Safety (LS), Collapse Prevention (CP)

- Hazard levels: Probability levels used to describe the earthquake intensity
  - Usually defined in terms of earthquake mean return periods or probability of exceedance (POE) during a certain duration
  - **Ex.**: 2475 years (2% POE in 50 years), 72 years (50% POE in 50 years)

- Performance objective: Objective of achieving a Performance Level under a given Hazard Level

Optimization-based methods

Tools: Earlier Studies

- Focused on single-objective optimization using gradient-based algorithms
  - These algorithms aim to minimize or maximize a real function by systematically choosing variables from within an allowed search space
  - Most commonly used types: linear and nonlinear programming, optimality criteria, and feasible directions
  - Computationally efficient due to rapid convergence rates
  - Require the existence of continuous objective functions and constraints in order to evaluate gradients, so the range of application is limited
- Objective function was almost exclusively selected as the initial cost or the material usage
- Several constraints (most often based on code provisions) were applied to determine the validity of designs
- Explicit formulations, which could be evaluated with little effort, were used for both the objective function and the constraints

Optimization-based methods

Tools: Modern Studies

- Most practical design problems in structural engineering require discrete representation of design variables (e.g. section sizes, reinforcement areas, …)
- The advent of numerical structural analysis methods has led to objective functions and/or constraints that are naturally discontinuous (e.g. EDPs)
- Researchers resorted to zero-order optimization algorithms that do not require existence of gradients or continuity of objective functions or constraints
- A class of zero-order optimization algorithms is the heuristic methods:
  - Genetic algorithms (GA)
  - Simulated annealing (SA)
  - Tabu search (TS)
  - Shuffled complex evolution (SCE)

Optimization-based methods

Tools: Modern Studies

Advantages of the heuristic methods:

- Can be adapted to solve any optimization problem with no requirements on the objectives and constraints
- Very effective in terms of finding the global minimum of highly nonlinear and/or discontinuous problems whereas gradient-based algorithms can easily be trapped at a local minimum

Criticism of the heuristic methods:

- Experience-based and depend on an improved version of basic trial-and-error
- Not based on a mathematical theory and there is no single heuristic optimization algorithm that is general for a wide class of optimization problems

Optimization-based methods

Tools: Modern Studies

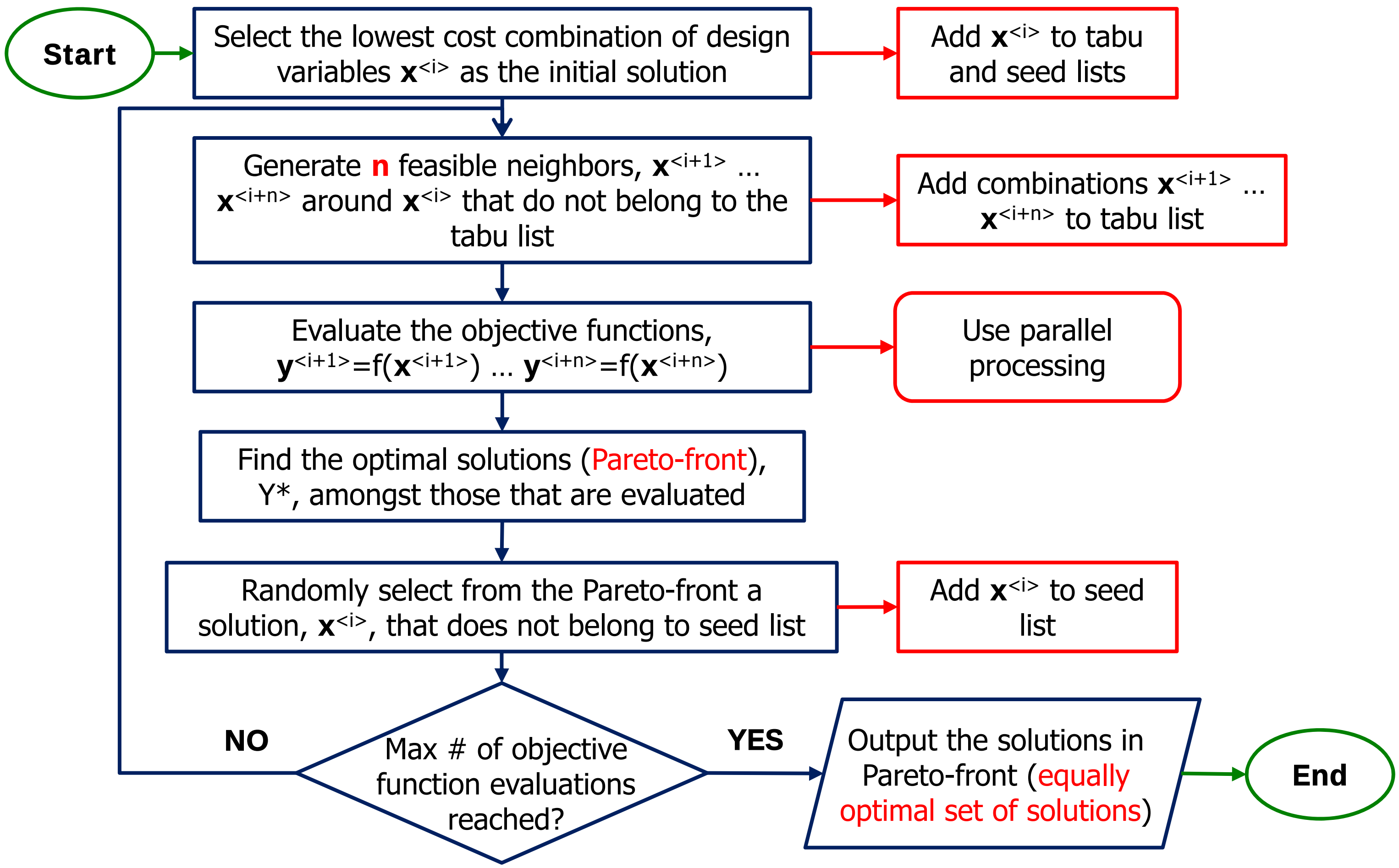
Tabu Search (Glover, 1989, 1990)

- Generally, used to solve combinatorial optimization problems (i.e. a problem of finding an optimum solution within a finite set of feasible solutions)
- Employs a neighborhood search procedure to sequentially move
  - From a combination of design variables $\mathbf{x}^{<i>}$, e.g. section sizes, reinforcement ratios, …, having a unique solution $\mathbf{y}^{<i>}$, e.g. MIDR, life cycle cost (LCC), …
  - To another in the neighborhood of $\mathbf{x}^{<i>}$ until some termination criterion has been reached ($\mathbf{x}^{<i>}$: seed point)
- Usually a portion of the neighboring points is selected randomly to prevent the algorithm to be trapped at a local minimum
- Keeps track of all previously employed $\mathbf{x}^{<i>}$ (tabu list & seed list), which are excluded from the set of neighboring points that are determined at each iteration
- Naturally lends itself to parallel processing, often needed to solve problems when evaluating the objective functions or the constraints is computationally costly

Optimization-based methods

Tools: Modern Studies

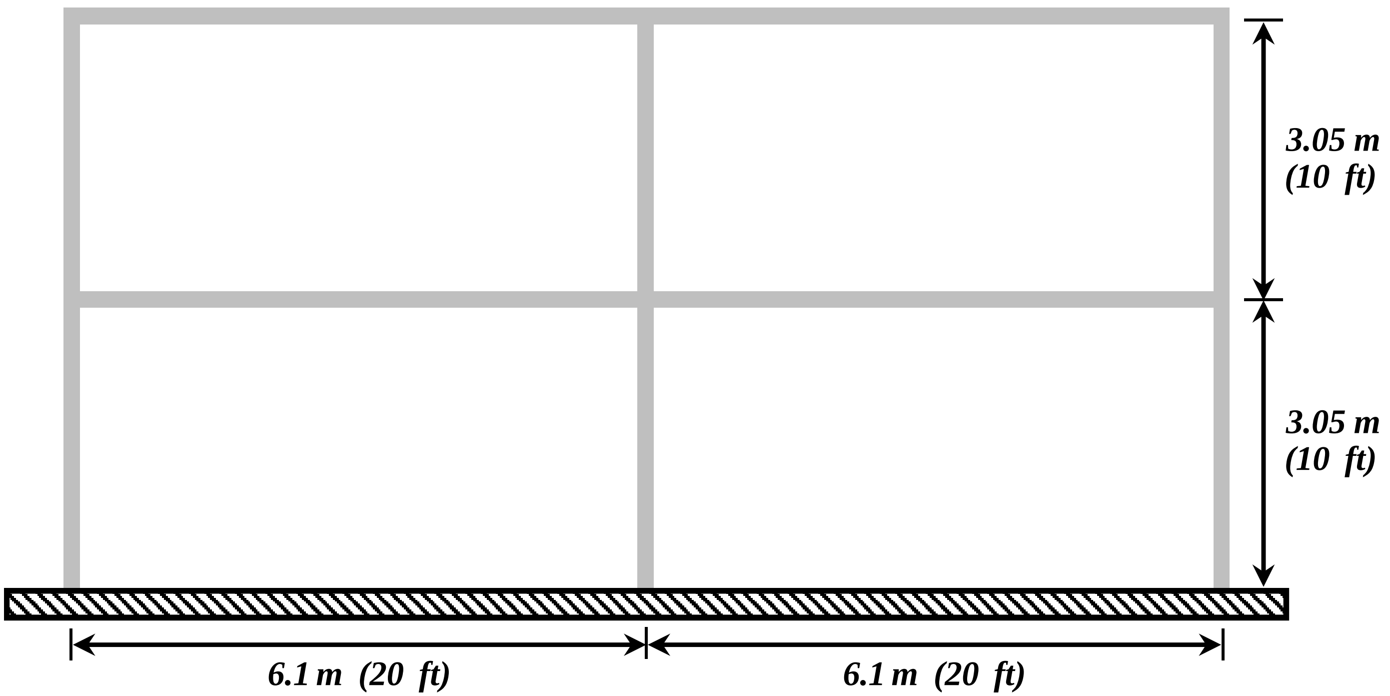
Tabu Search (Glover, 1989, 1990)



Optimization-based methods

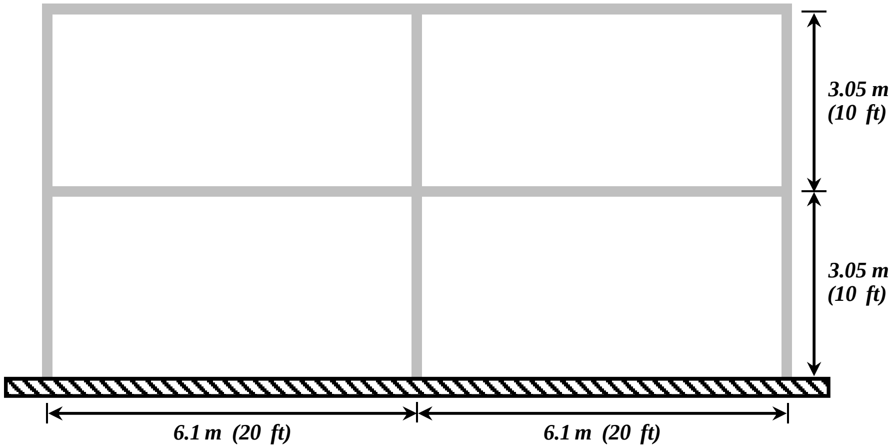
Illustrative Example:

Optimization of a two-story two-bay RC frame



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame



Design Variables (**x**):

- Column Reinforcement Ratio (%)
- Beam Reinforcement Ratio (%)
- Width of Exterior Columns (mm)
- Width of Interior Columns (mm)
- Depth of Columns (mm)
- Depth of Beams (mm)
- Width of Beams (mm)

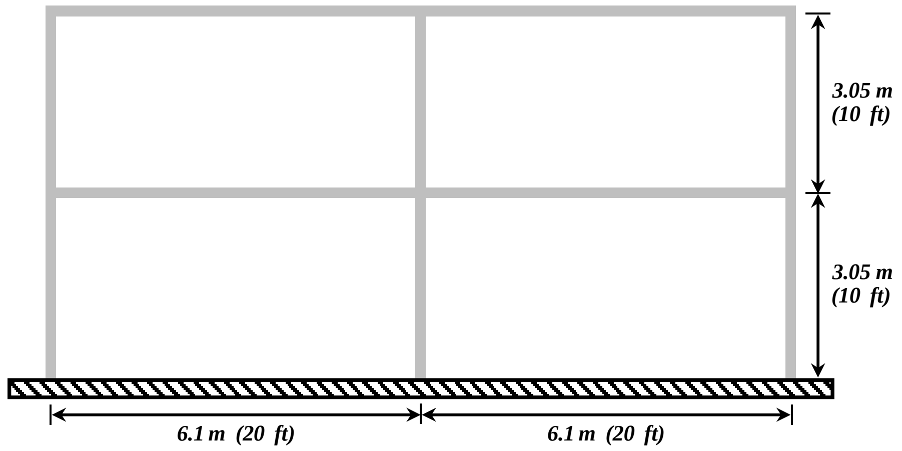
Objective Functions [**y**=f(**x**)]:

- Initial Cost
- Expected value of Life-cycle Cost (LCC)
- Maximum Interstory Drift Ratio (MIDR)

Target: Minimize the objective functions

Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame



Search Space

Design Variables	Minimum	Maximum	Increment
Column Reinforcement Ratio (%)	1.0	3.0	0.5
Beam Reinforcement Ratio (%)	1.0	3.0	0.5
Width of Exterior Columns (mm)	304.8	508	50.8
Width of Interior Columns (mm)	355.6	558.8	50.8
Depth of Columns (mm)	304.8	457.2	50.8
Depth of Beams (mm)	406.4	558.8	50.8
Width of Beams (mm)	304.8	406.4	50.8

Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

Objective Functions [y=f(x)]

- Initial Cost (C_0):
  - C_0 = Cost of (Steel + Concrete + Formwork + Labor)
  - Estimated according to 2011 Building Construction Cost Data
- Expected Value of Life Cycle Cost (E[LCC]):
  - LCC is a random quantity due to various sources of uncertainty including
    - o Ground motion variability,
    - o Modeling error (see next slide),
    - o Unknown material properties
  - The expected LCC of a structure, incorporating both aleatory uncertainty due to ground motion variability and epistemic uncertainty due to modeling error, is expressed as follows:

$$E[LCC] = C_0 + \int_0^L E[C_{SD}]\left(\frac{1}{1+\lambda}\right)^t dt = C_0 + \alpha L E[C_{SD}]$$

See Slide 21

Optimization-based methods

Modeling error

Courtesy of Prof. S. Mahin

41 expert teams participated

Full-scale 1D tests of circular column - Jose Restrepo, PI (PEER, Caltrans, UNR, FHWA, NEES@UCSD, NEEScomm & NSF)

Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

- Expected Value of Life Cycle Cost (E[LCC]):

$$E[LCC] = C_0 + \int_0^L E[C_{SD}] \left(\frac{1}{1+\lambda} \right)^t dt$$

Expected seismic damage cost

(Assumed to be governed by a Poisson process)

Poisson process: A stochastic process where time between pairs of consecutive events has exponential distribution & these inter-arrival times is assumed independent of other inter-arrival times.

$$E[LCC] = C_0 + \alpha L\, E[C_{SD}] \qquad \alpha = [1 - \exp(-qL)]/qL \qquad q = \ln(1+\lambda)$$

L: Life span

λ: Annual discount rate

$$E[C_{SD}] = \sum_{i=1}^{N} C_i p_i$$

Probability of i^{th} damage state:

$$p_i = p(\Delta_D > \Delta_{C,i}) - p(\Delta_D > \Delta_{C,i+1})$$

D: demand, C: capacity

See Next Slide for example

Cost for i^{th} damage state:

- 30% IO-LS
- 70% LS-CP
- 100% CP

Total number of considered damage-states:

- IO-LS (state between Immediate Occupancy and Life Safety)
- LS-CP (state between Life safety and the Collapse Prevention)
- CP (Collapse Prevention)

Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

SAC/FEMA equation:

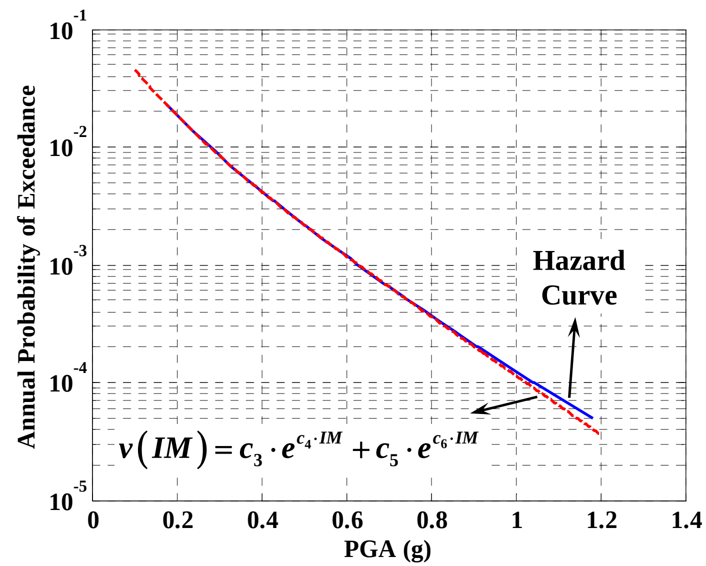
$$p\left(\Delta_D > \Delta_{C,i}\right) = \int_0^\infty p\left(\Delta_D > \Delta_{C,i} \mid IM = im\right) \left|\frac{d\nu(IM)}{dIM}\right| dIM$$

Conditional probability of demand being greater than the capacity given the ground motion intensity [**See next slide**]

Slope of the hazard curve: Possible to obtain analytically by fitting a function to the curve

$$\nu(IM) = c_3 \cdot \exp(c_4 \cdot IM) + c_5 \cdot \exp(c_6 \cdot IM)$$

$$\frac{d\nu(IM)}{dIM} = c_3 \cdot c_4 \cdot \exp(c_4 \cdot IM) + c_5 \cdot c_6 \cdot \exp(c_6 \cdot IM)$$



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

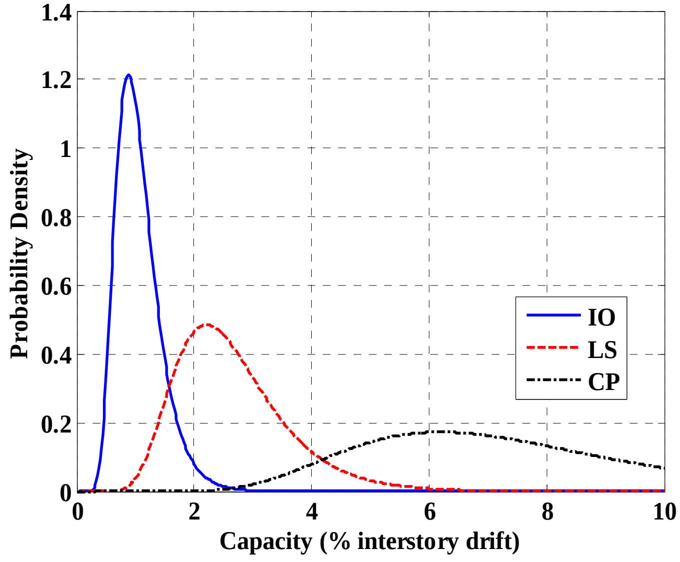
$$p\left(\Delta_D > \Delta_{C,i} \mid IM = im\right) = \int_0^\infty p\left(\Delta_D > \delta \mid IM = im\right) f_{C,i}(\delta)\, d\delta$$

See next slide

Probability density function for structural capacity for the i^{th} damage state

lognormal distribution with logarithmic mean & the standard deviation $\Delta_{C,i}$ & β_C, respectively.

The uncertainty in capacity represented with β_C accounts for factors such as modelling errors & variations in material properties



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

$$p\left(\Delta_D > \delta \mid IM = im\right) = 1 - \Phi\left[\frac{\ln(\delta) - \lambda_{D|IM=im}}{\beta_D}\right]$$

mean of the natural logarithm of the earthquake demand (function of ground motion intensity) [$\lambda_D = \ln(\mu_D)$]

Logarithmic standard deviation of the corresponding normal distribution of the earthquake demand

Standard normal cumulative distribution

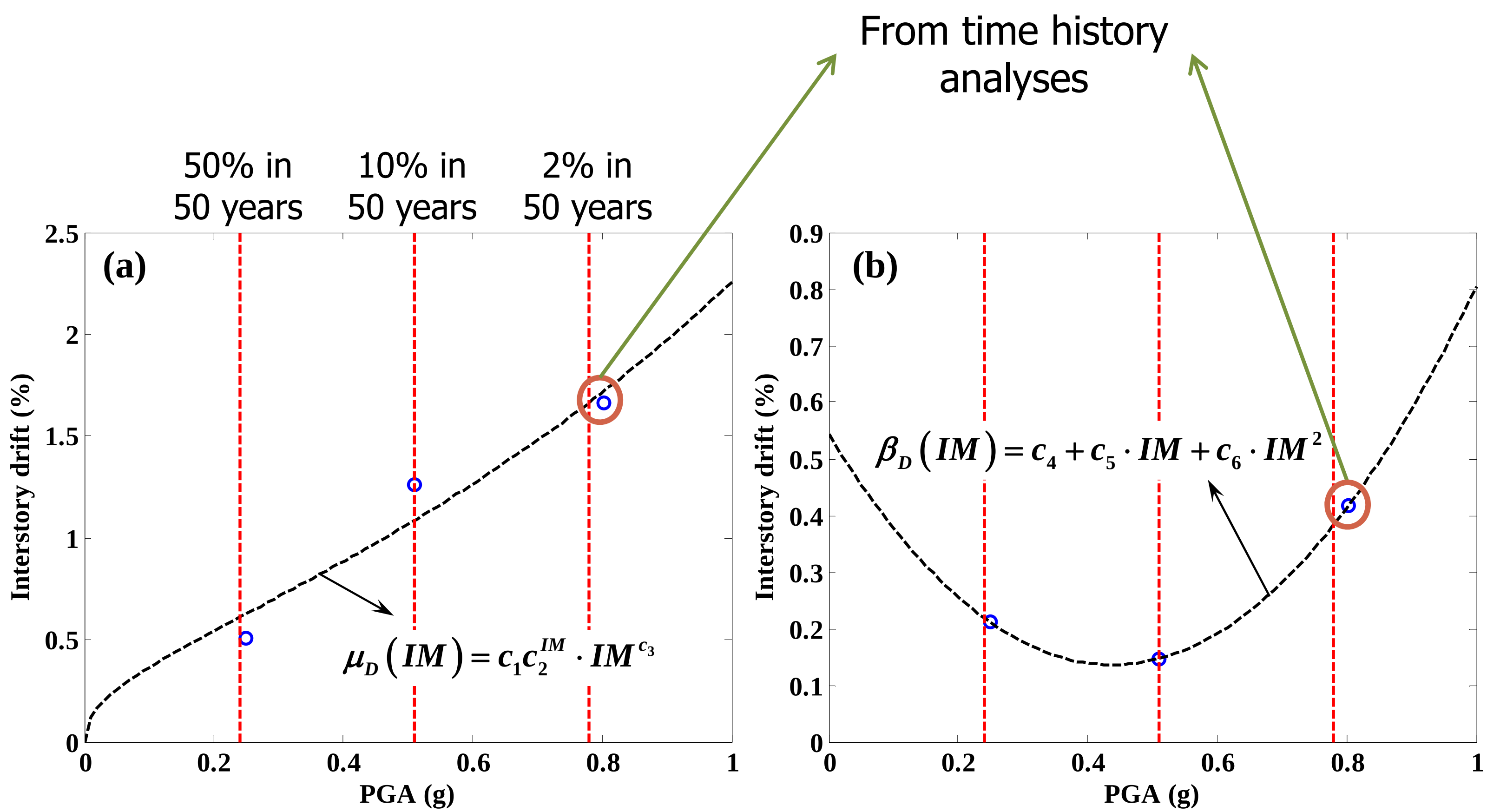
It is possible to describe μ_D & β_D as continuous functions of the ground motion intensity (Aslani and Miranda, 2005)

$$\mu_D(IM) = c_1 c_2^{IM} IM^{c_3} \qquad \beta_D(IM) = c_4 + c_5 \cdot IM + c_6 \cdot IM^2$$

Constants c_1–c_3 & c_4–c_6 are determined by fitting a curve to the mean & logarithmic standard deviation values obtained from time history analyses of the analytical model

Optimization-based methods

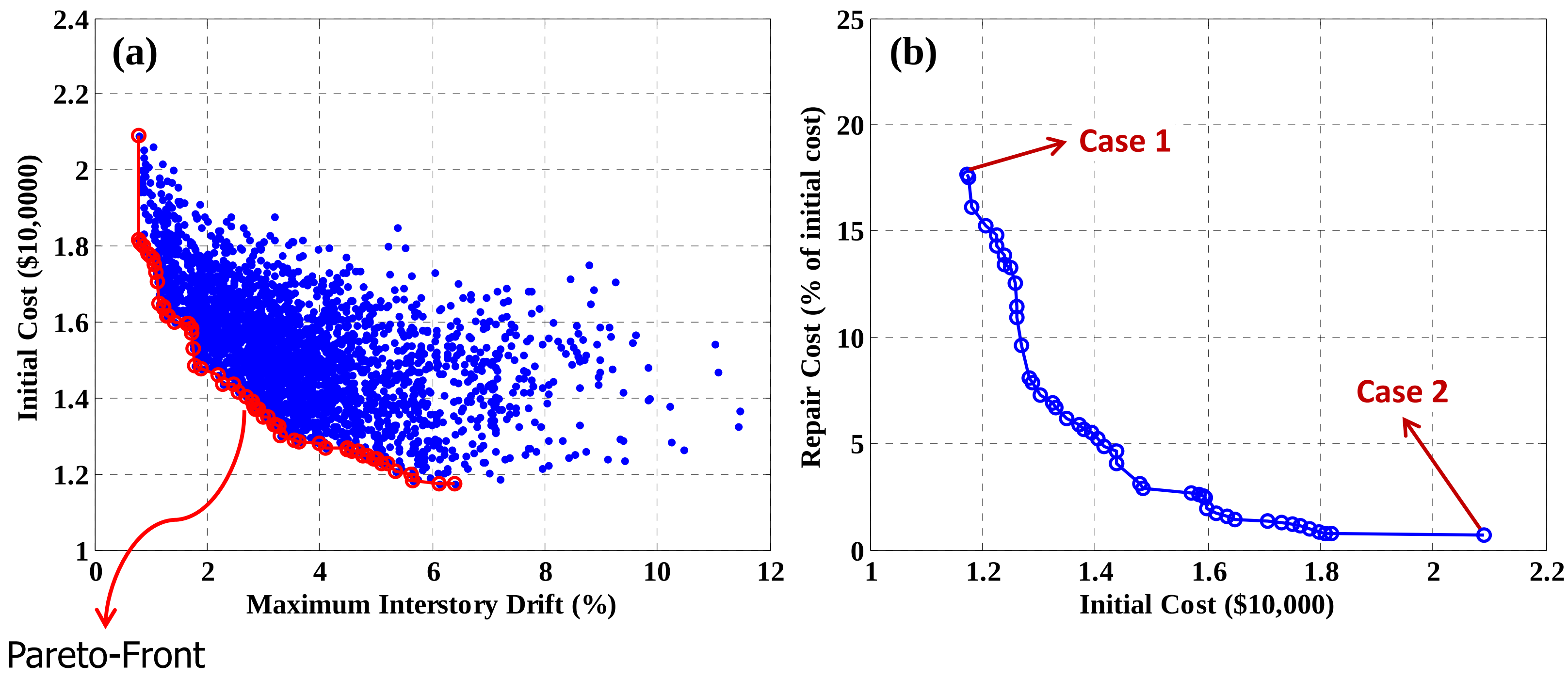
Illustrative Example: Optimization of a two-story two-bay RC frame



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

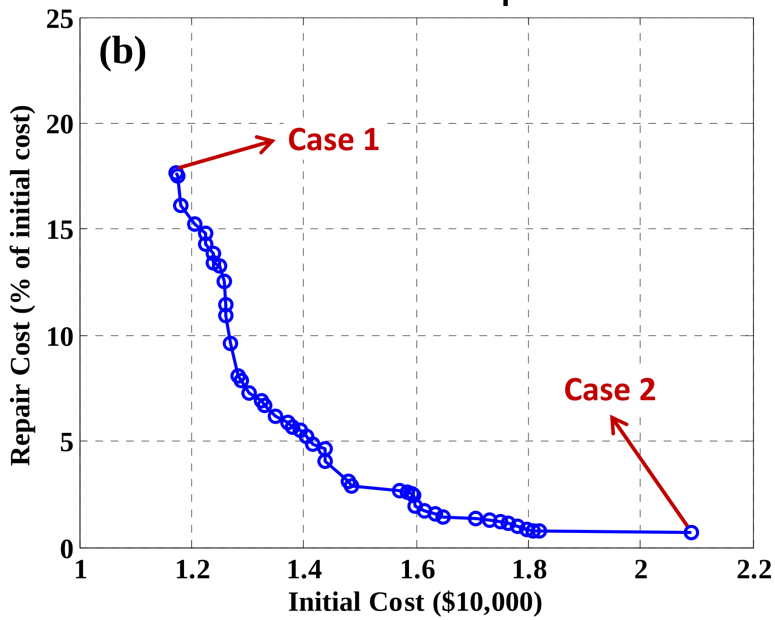
Optimization Results with TS algorithm



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

Optimization Results with TS algorithm



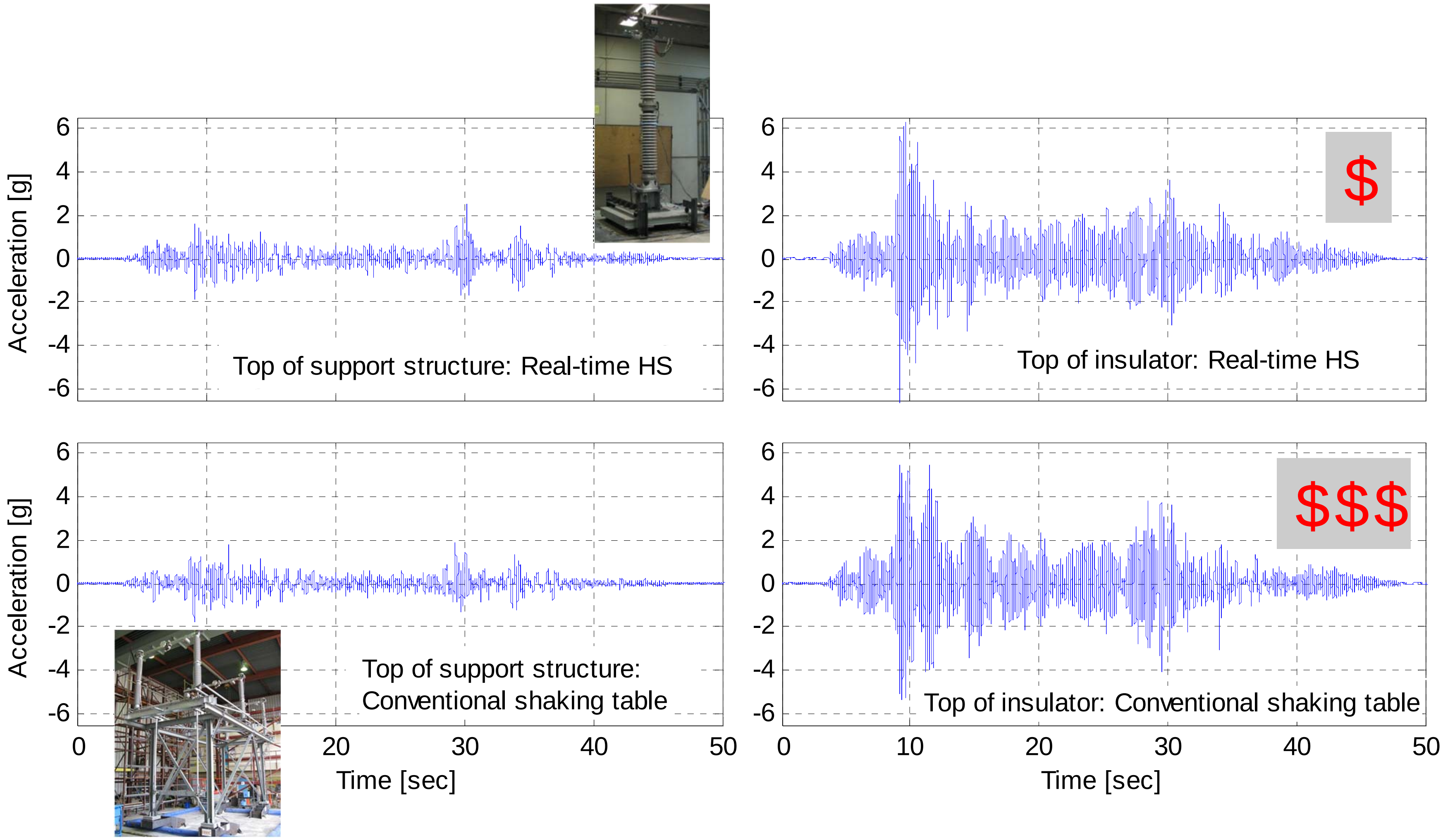
	Case 1	Case 2
Column Reinforcement Ratio (%)	1.5	3.0
Beam Reinforcement Ratio (%)	1.0	3.0
Width of Exterior Columns (mm)	304.8	508
Width of Interior Columns (mm)	355.6	558.8
Depth of Columns (mm)	304.8	457.2
Depth of Beams (mm)	406.4	558.8
Width of Beams (mm)	304.8	406.4

- ❑ Representation of equivalently optimal solutions using Pareto-optimality is very useful for decision makers
- ❑ It provides the decision maker with flexibility to choose among a set of equivalently optimal solutions depending on the requirements of the project
- ❑ The extent to which the desired structural performance would be satisfied by a selected alternative can be easily observed

Traditional earthquake design is not sufficient but *necessary*. Future exercise: Check design of cases 1 & 2 with requirements of seismic codes, e.g. strong column-weak beam, shear failure prevention, …, etc.)

Optimization-based methods

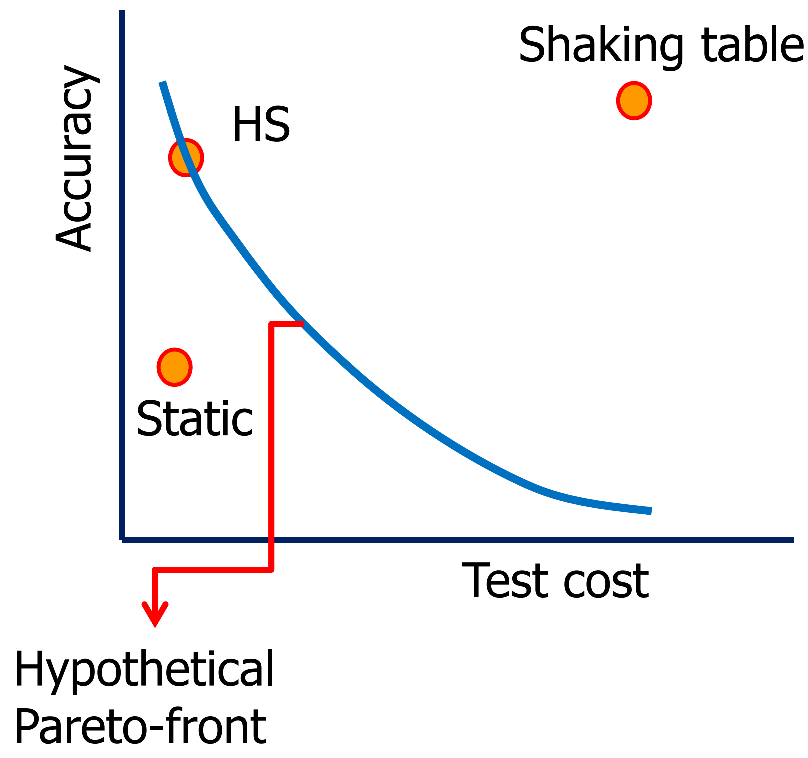
An optimization problem related to yesterday's HS workshop



Optimization-based methods

An optimization problem related to yesterday's HS workshop

Qualitative justification of Hybrid Simulation
with an Optimization Technique



Non optimization-based methods

Two available non-optimization-based approaches

- ❑ Krawinkler et al. (2006)
- ❑ Franchin and Pinto (2012)

Krawinkler et al. (2006):

- Can not be considered as a fully probabilistic design procedure
- More in line with first-generation PBEE procedures
- Iteratively enforces satisfaction of two performance objectives associated with 50/50 and 2/50 hazard levels in terms of cost
- Makes use of median incremental dynamic analysis (IDA) curves [Vamvatsikos and Cornell 2002] to relate the hazard levels with the corresponding EDPs, as well as the average loss curves
- The design variables are the fundamental period $\mathbf{T_1}$ and the base shear ratio η (ratio of base shear to the weight of the structure).
- Requires a prior production of design-aids in the form of alternative median IDA curves for different values of the design variables.

Non optimization-based methods

Two available non-optimization-based approaches

- ❑ Krawinkler et al. (2006)
- ❑ Franchin and Pinto (2012)

Franchin and Pinto (2012):

- ❑ Fully probabilistic
- ❑ Employs constraints formulated explicitly in terms of MAF of exceedance of chosen performance-levels
- ❑ Can be considered as an approximate method relying on the validity of the following:
  - Closed-form expression for MAF of exceedance of a limit-state [Cornell et al., 2002]
  - Equal-displacement rule [Veletsos and Newmark, 1960]
- ❑ Difference with respect to the optimization approaches: Method produces a solution that is feasible, i.e. that complies with constraints, but not necessarily optimal
- ❑ Extension to include an objective function related to, e.g. minimum cost, is possible within the same framework

Thank you