PEER PBEE Formulation

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Probabilistic Performance-based Earthquake Engineering, University of Minho, Guimarães, Portugal, October 3-4, 2012

Outline

1. Introduction
2. Hazard Analysis
3. Structural Analysis
4. Damage Analysis
5. Loss Analysis
6. Combination of Analyses

Introduction

- **Traditional earthquake design (TED) philosophy:**
  - ➢Prevent damage in low-intensity EQ
  - ➢Limit damage to repairable levels in medium-intensity EQ
  - ➢Prevent collapse in high-intensity EQ

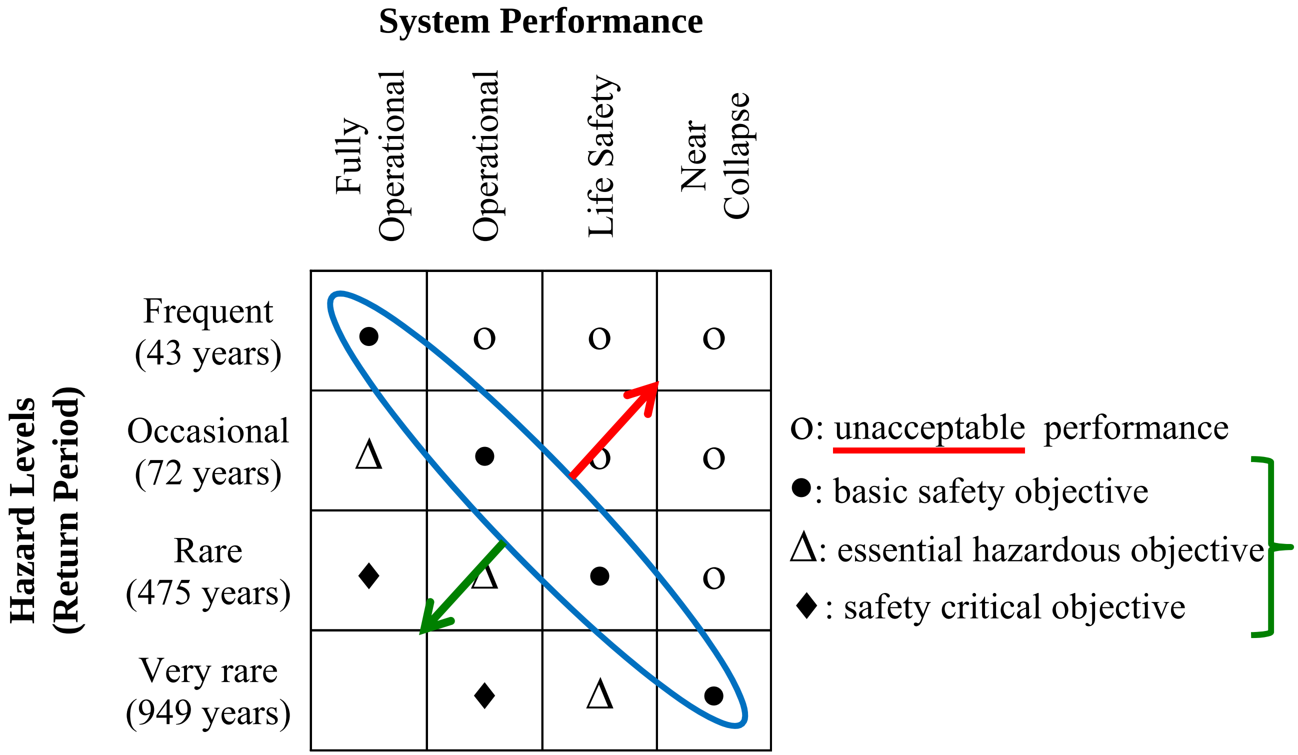
- **TED is necessary but not sufficient as evidenced by:**
  - ➢1994 Northridge and 1995 Kobe earthquakes (initial realizations)

    Unacceptably high amount of damage, economic loss due to downtime, and repair cost of structures
  - ➢2009 L'Aquila and 2010 Chile earthquakes (recent evidences)
    - A traditionally designed hospital building evacuated immediately after L'Aquila EQ, while ambulances were arriving with injured people
    - Some hospitals evacuated due to non-structural damage and damage to infill walls after Chile EQ
    - Some of the residents rejects to live in their homes anymore despite satisfactory performance according to the available codes

Introduction

- **First generation PBEE methods:**

Improvement to Traditional Earthquake Design by introducing "<u>Performance Objectives</u>": Achieve a desired "System Performance" at a given "Seismic Hazard"



System Performance

Hazard Levels (Return Period)	Fully Operational	Operational	Life Safety	Near Collapse
Frequent (43 years)	●	o	o	o
Occasional (72 years)	Δ	●	o	o
Rare (475 years)	♦	Δ	●	o
Very rare (949 years)		♦	Δ	●

o: unacceptable performance
●: basic safety objective
Δ: essential hazardous objective
♦ : safety critical objective

Introduction

- **First generation PBEE methods - Shortcomings:**

  - Deterministic evaluation of performance: Lack of consideration of uncertainty
  - Evaluation on the element level: Lack of consistency in the determination of the relationships between engineering demands and component performance criteria
  - Evaluation on the element level: Not tied to global system performance
  - Results specific to engineers: Reduced contribution of stakeholders in the decision process

Introduction

- **Pacific Earthquake Engineering Research (PEER) Center PBEE:**

❖ Improvement of first generation PBEE by introducing:

- ✓ Calculation of performance in a rigorous probabilistic manner: Consideration of uncertainty
- ✓ Performance definition with decision variables which reflect the global system performance
- ✓ Performance definition with decision variables in terms of the direct interest of various stakeholders

X Shortcoming: Mostly used by academia with ***little*** attention from practicing engineers

Introduction

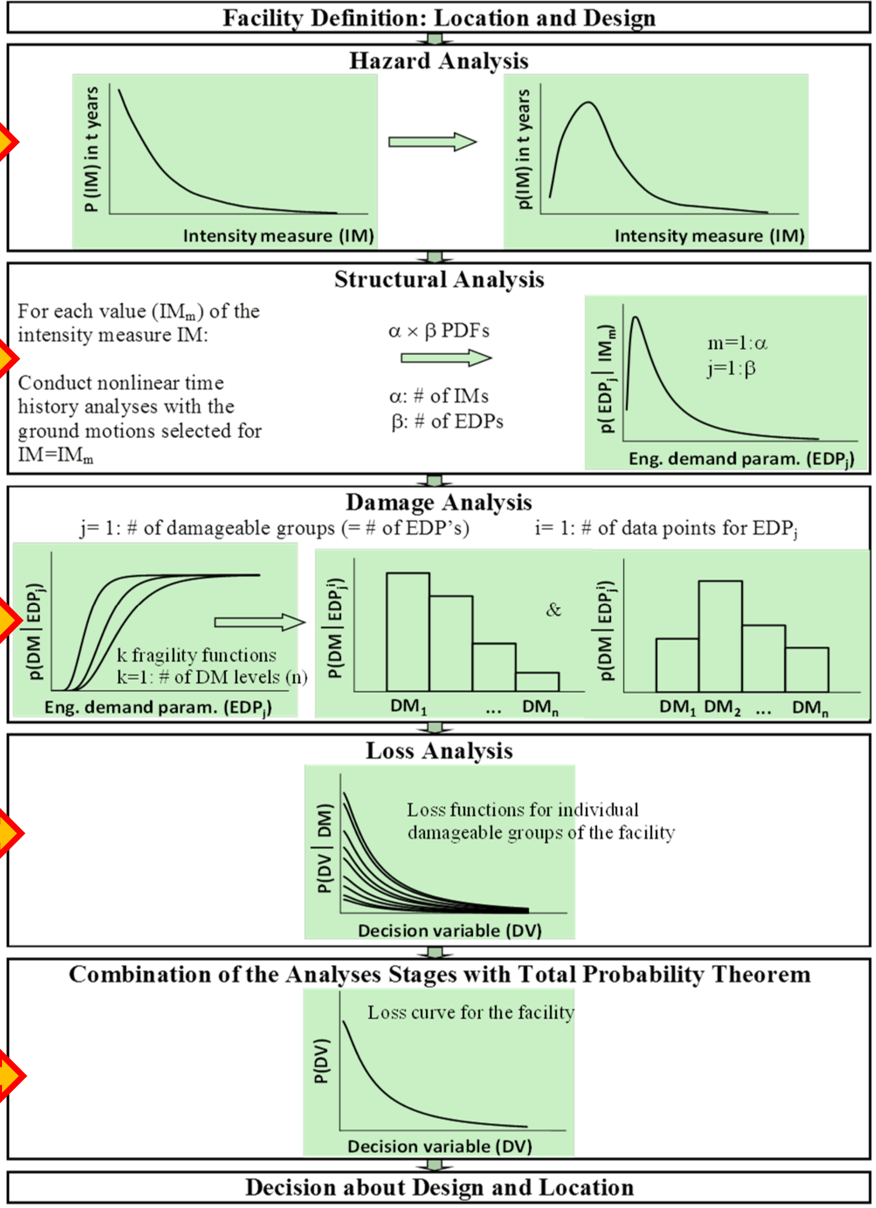
- **PEER PBEE (Revisited):**

  - Gaining popularity of probabilistic Performance-Based Engineering Design (PBED) methods
  - PBED methods likely to be used for standard design codes in the near future
  - Necessity to find paths for popularization of the method within the practicing structural engineering community
  - **Objective**: Explain PEER PBEE methodology in a simplified manner to reach the broader engineering community

PEER PBEE Formulation

- **Hazard Analysis**: Earthquake hazard during the lifecycle of a building (uncertainty in fault locations, magnitude-recurrence rates, level of attenuation, etc.)
- **Structural Analysis**: Response of the structure to the earthquake hazard (uncertainty in ground motion type, material properties, damping, etc.)
- **Damage Analysis**: Level of damage corresponding to the response of the structure (uncertainty in the damage pattern, history, capacity, etc.)
- **Loss Analysis**: Value of a decision variable (DV, e.g. economic loss) corresponding to damage (uncertainty in damage distribution, variation of components resulting in same damage level, etc.)

End Product: Due to the different sources of uncertainty, there is no single deterministic value of DV. Instead, there are multiple values of DV with varying probability.



Facility Definition: Location and Design

Hazard Analysis

Structural Analysis

For each value (IM_m) of the intensity measure IM:

Conduct nonlinear time history analyses with the ground motions selected for $IM=IM_m$

$\alpha \times \beta$ PDFs

α: # of IMs

β: # of EDPs

Damage Analysis

j= 1: # of damageable groups (= # of EDP's)

i= 1: # of data points for EDP_j

Loss Analysis

Combination of the Analyses Stages with Total Probability Theorem

Decision about Design and Location

Hazard Analysis

- First analysis stage in PEER PBEE formulation

- A natural hazard is a threat of a naturally occurring event that will have a negative effect on people or the environment:
  - Earthquakes
  - Volcanoes
  - Hurricanes
  - Landslides
  - Floods or droughts
  - Wildfires

- PEER PBEE considers earthquake hazard (seismic hazard)

Hazard Analysis

- <u>Uncertainty in seismic hazard</u>:

  a. Potential fault locations

  b. Magnitude-recurrence rates

  c. Level of attenuation

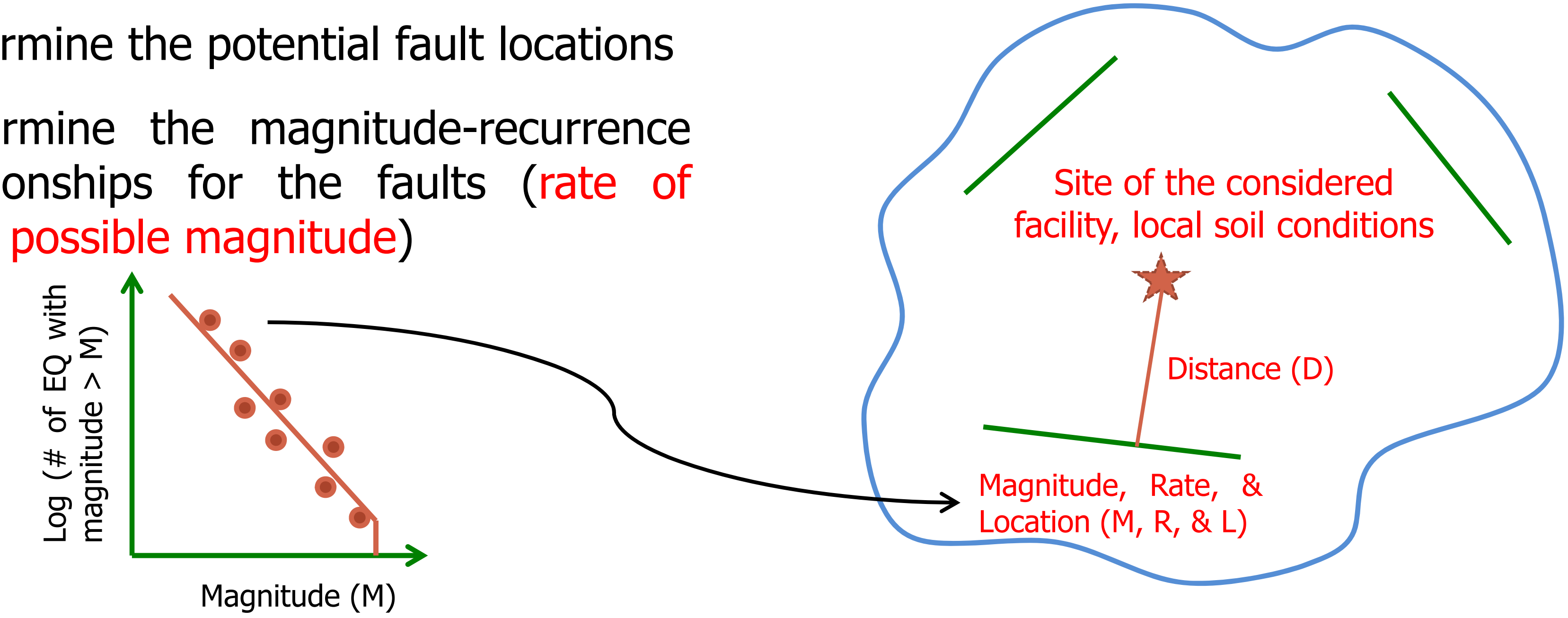
- Deterministic Seismic Hazard Analysis (Limited uncertainty consideration: only item "**c**" above)

- Probabilistic Seismic Hazard Analysis (Complete uncertainty consideration → <u>Preferred method</u>)

Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

1. Determine the potential fault locations
2. Determine the magnitude-recurrence relationships for the faults (rate of each possible magnitude)



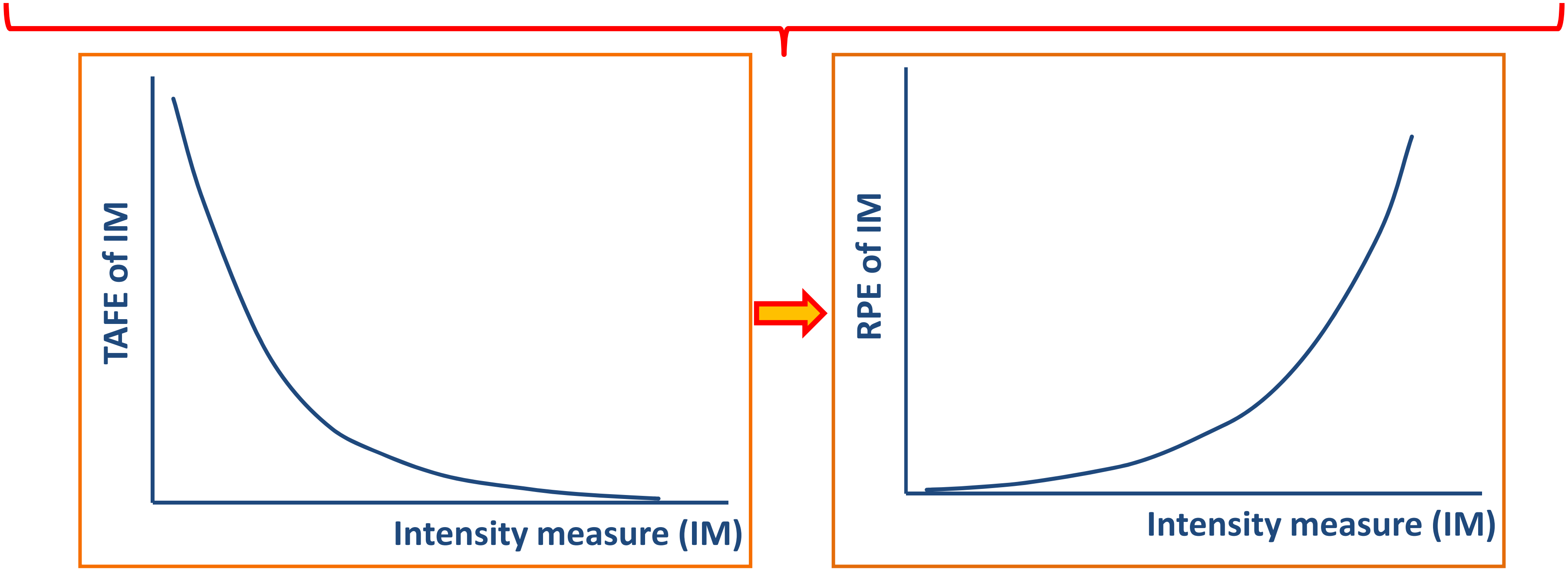
3. For all the potential earthquake scenarios (M, R, & L):
   - ➢ Using ground motion prediction equations: Calculate the mean and standard deviation (μ & σ) of intensity measure (IM) as a function of (M, D)
   - ➢ Determine the probability distribution function (PDF) and probability of exceedance (POE) of IM using μ & σ
   - ➢ Multiply POE with R to determine annual frequency of exceedance (AFE) of IM

Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

4. Sum AFE from all scenarios to obtain the total annual frequency of exceedance (TAFE) of IM

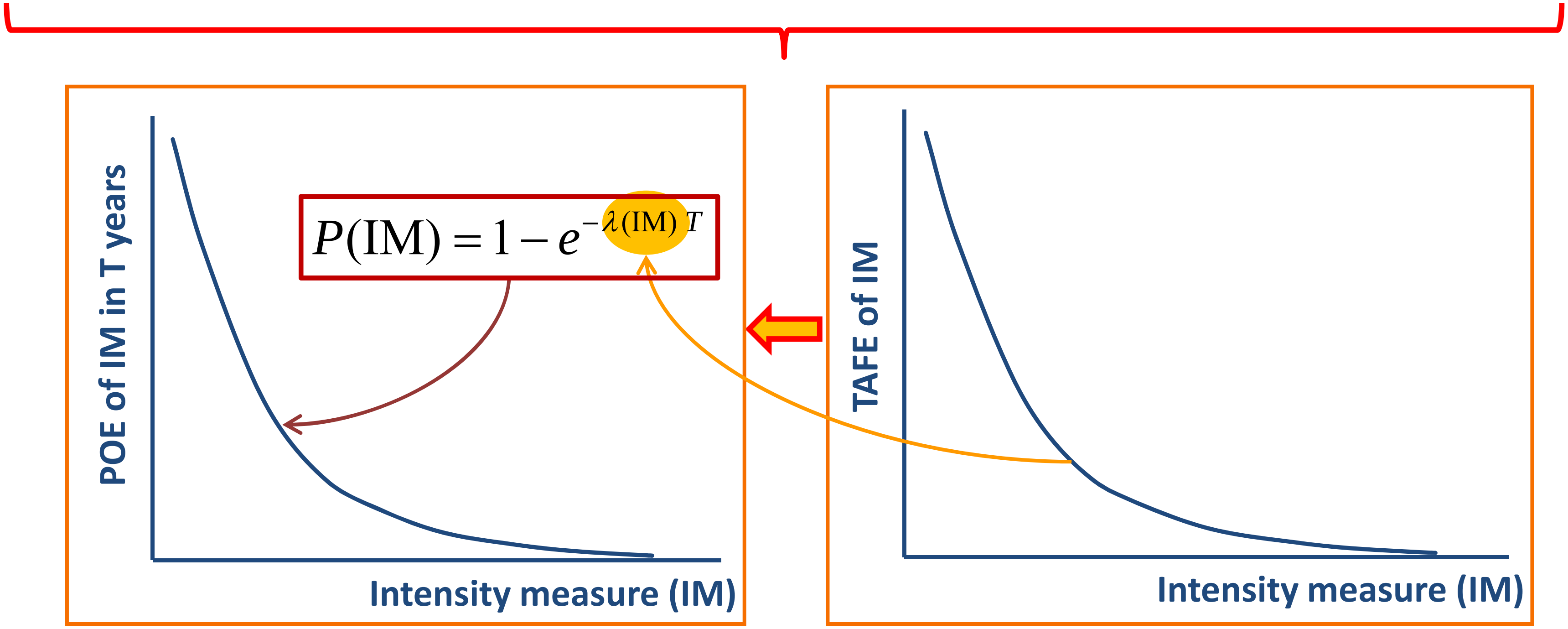
   An easier way of representation of TAFE: Return period of exceedance, RPE = 1/TAFE



Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

5. From Poisson’s model, calculate POE of IM in T years from TAFE



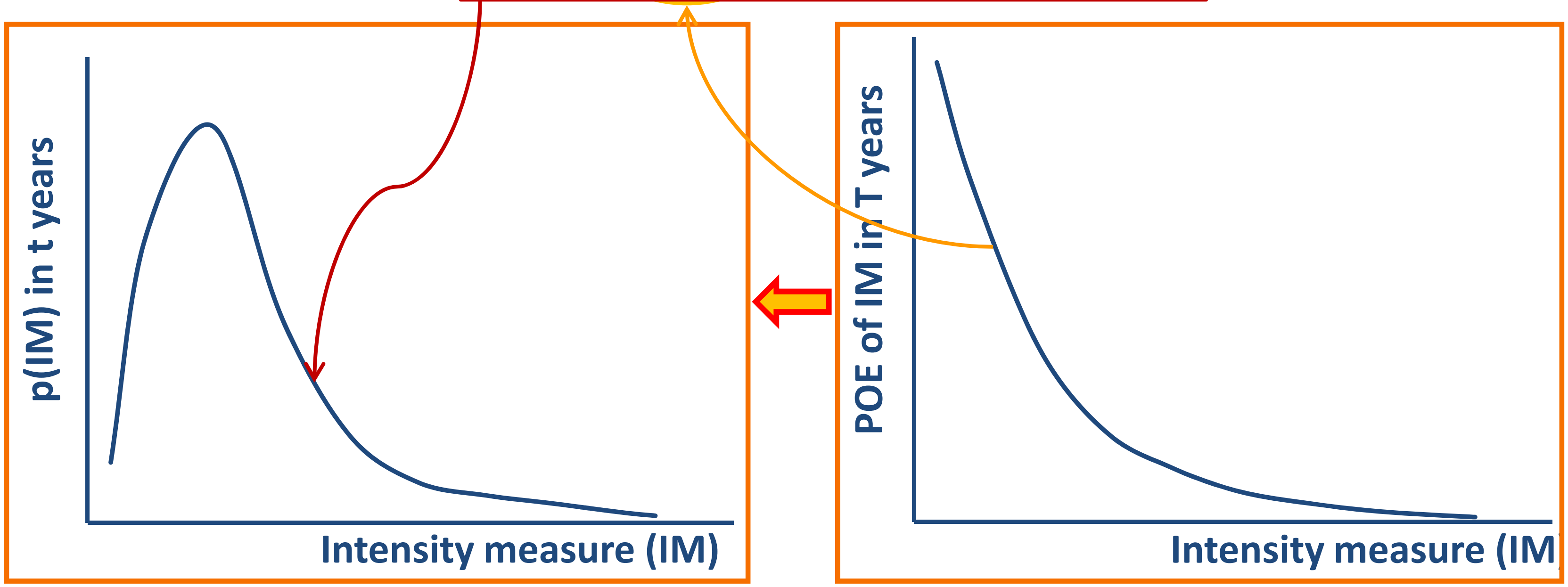
$$P(\mathrm{IM}) = 1 - e^{-\lambda(\mathrm{IM})\,T}$$

Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

6. Calculate probability of IM in T years from POE

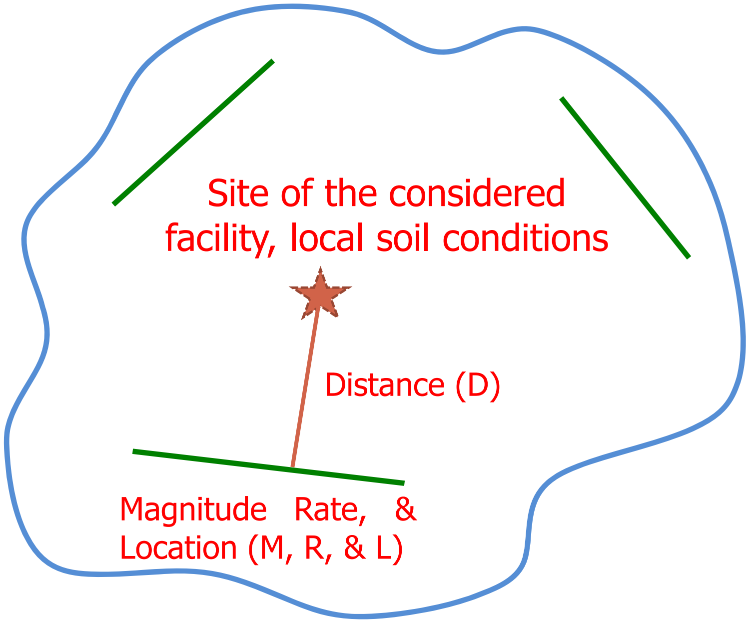
```
for m = 1 : # of IM data points
    $p(IM_m) = P(IM_m)$    if m = # of IM data points
    $p(IM_m) = P(IM_m) - P(IM_{m+1})$    otherwise
```



Hazard Analysis

Deterministic Seismic Hazard Analysis (DSHA)

1. and 2. as PSHA
3. For one or only few (generally the most critical) of the potential earthquake scenarios (M, R, & L)
   - Determine the value of intensity measure (IM) as a function of (M, D)
   - Inherent consideration of uncertainty due to the probabilistic nature of ground motion prediction equations



Hazard Analysis

- **Outcome of hazard analysis**: Probability of exceedance (POE) and probability (p) of Intensity Measure (IM)
- **Commonly used IMs**:
  - Peak ground acceleration [PGA]
  - Peak ground velocity [PGV]
  - Spectral acceleration at fundamental period [Sa(T_1)]

Reason of common use: Ground motion predictions available

- **Alternatives for IM** [e.g., Tothong and Cornell (2007)]:
  - Inelastic spectral displacement
  - Inelastic spectral displacement with a higher-mode factor

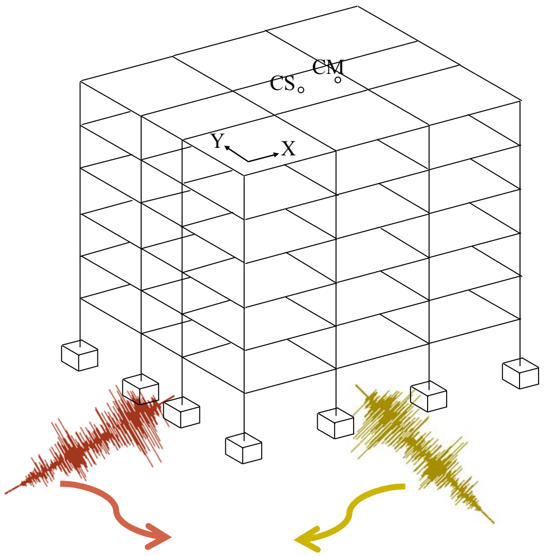
Hazard Analysis

- **Selection of ground motion time histories**: Compatible with the hazard curve for each intensity level (i.e. each IM value)

  - Adequate number of GMs to provide meaningful statistical data in the structural analysis phase
  - GMs compatible with the magnitude and distance pair which dominates the hazard
  - Use of unscaled GMs whenever possible
  - Separation of unscaled ground motions into bins: Performed once and used for consecutive cases

Structural Analysis

- ❖ Second analysis stage in PEER PBEE Formulation
- ❖ A computational model of the structure:



GMs from hazard analysis
(*uncertainty in GM characteristics*)

Uncertainty in
- Mass (e.g. variation in live load)
- Damping (e.g. epistemic uncertainty in damping models)
- Material characteristics (e.g. strength, ultimate strain)

- ❖ Nonlinear time history simulations with ground motions from hazard analysis

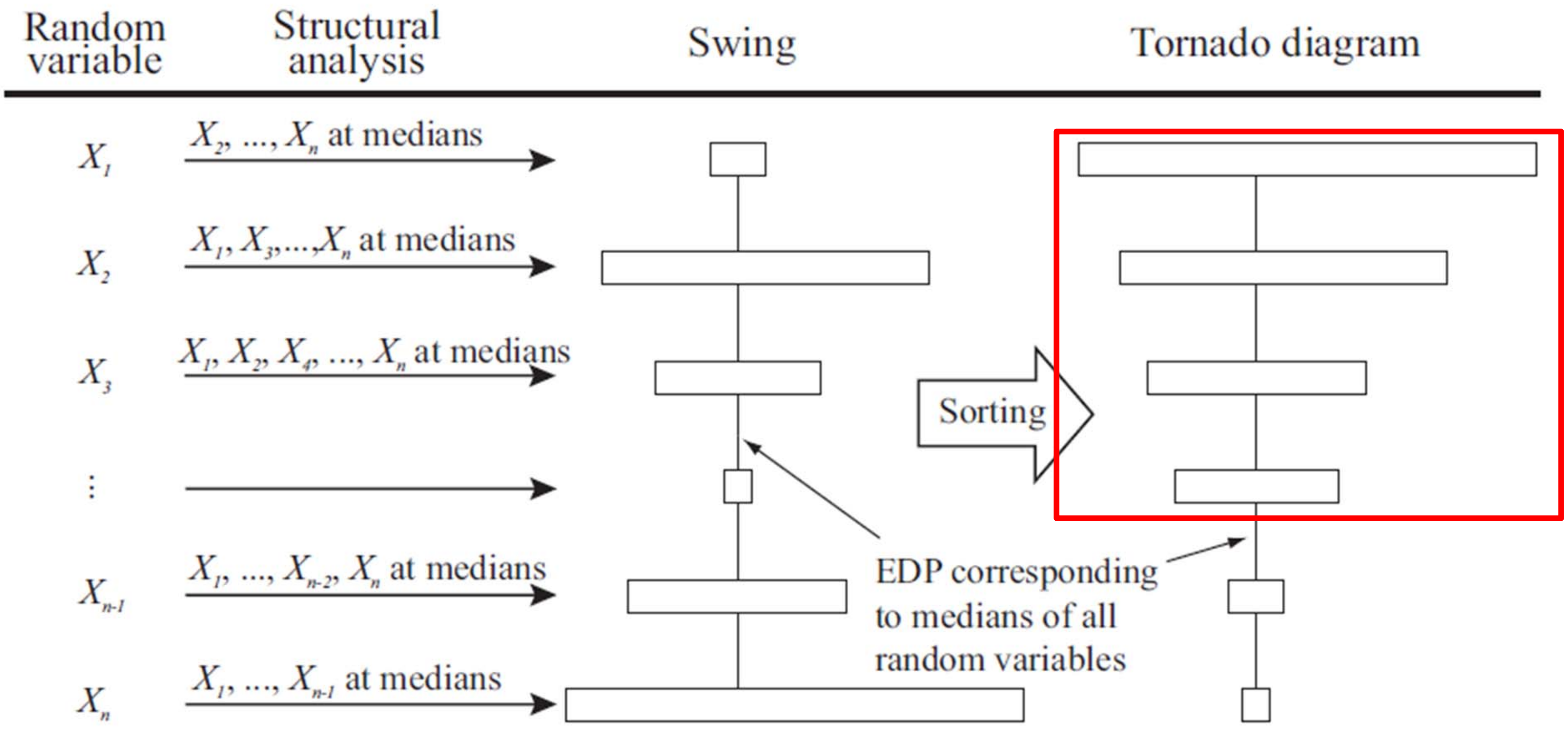
Structural Analysis

❖ **Potential variables in analyses:**

- Ground motion
- Mass
- Damping ratio
- Damping model
- Strength
- Modulus of elasticity
- Ultimate strain

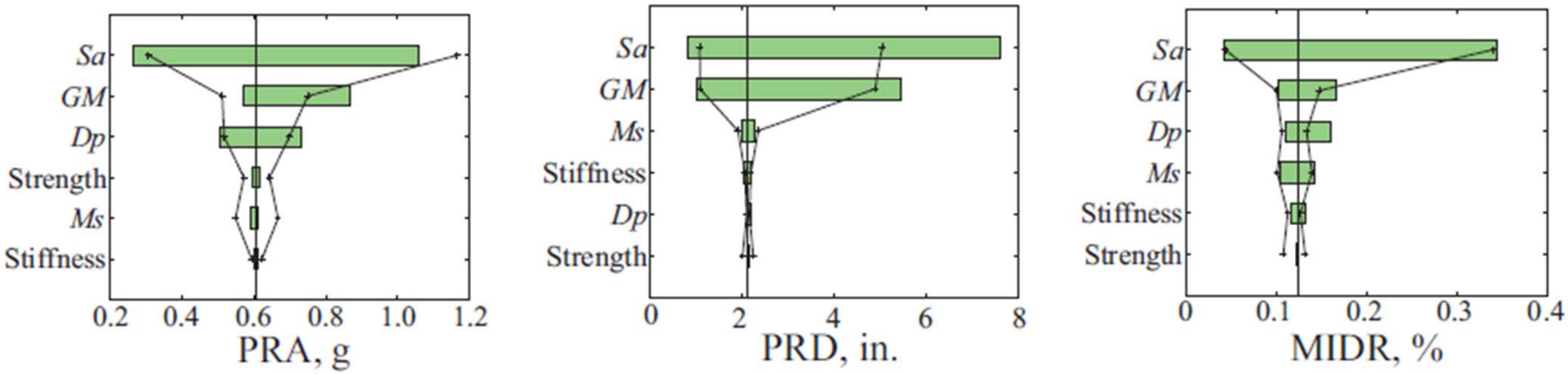
Structural Analysis

- ❖ **Determination of** **important** **variables:** Tornado diagram analysis (Lee and Mosalam, 2006)



Structural Analysis

- ❖ **Determination of important variables:** Tornado diagram analysis (Lee and Mosalam, 2006)

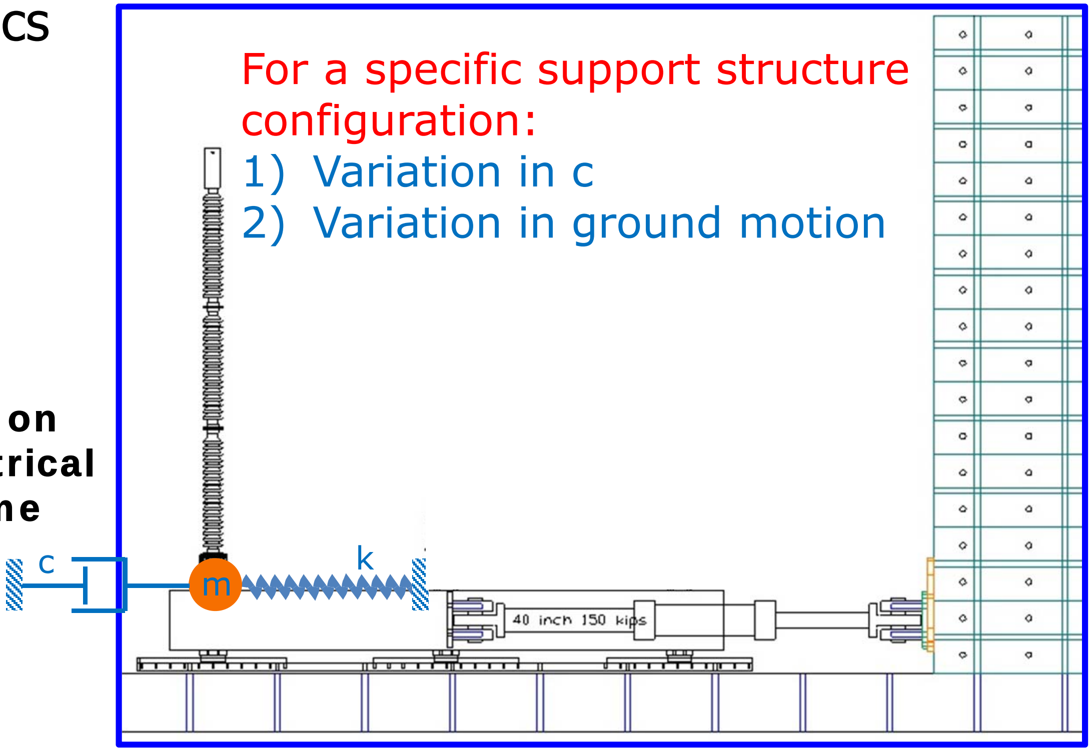


- ❖ Determine the variables with negligible effect on the structural response variability and reduce the number of simulations by eliminating unnecessary sources of uncertainties

Structural Analysis

- ❖ Remember Hybrid Simulation (from yesterday's workshop)
  - In some cases, hybrid simulation can be an alternative to the nonlinear time history simulations
  - For example, elimination of the simulations for the uncertainties in material characteristics

Investigation of the Effect of support structure properties on the seismic response of electrical insulator posts using real-time hybrid simulation (RTHS)



For a specific support structure configuration:

1) Variation in c
2) Variation in ground motion

Structural Analysis

- **Structural analysis outcome**: Engineering Demand Parameter (EDP)
- Local parameters: e.g. element forces & deformations
- Global parameters: e.g. floor acceleration & interstory drift
- **Different EDPs for different damageable groups**:
  - Axial or shear force in a non-ductile column
  - Plastic rotations for ductile flexural behavior

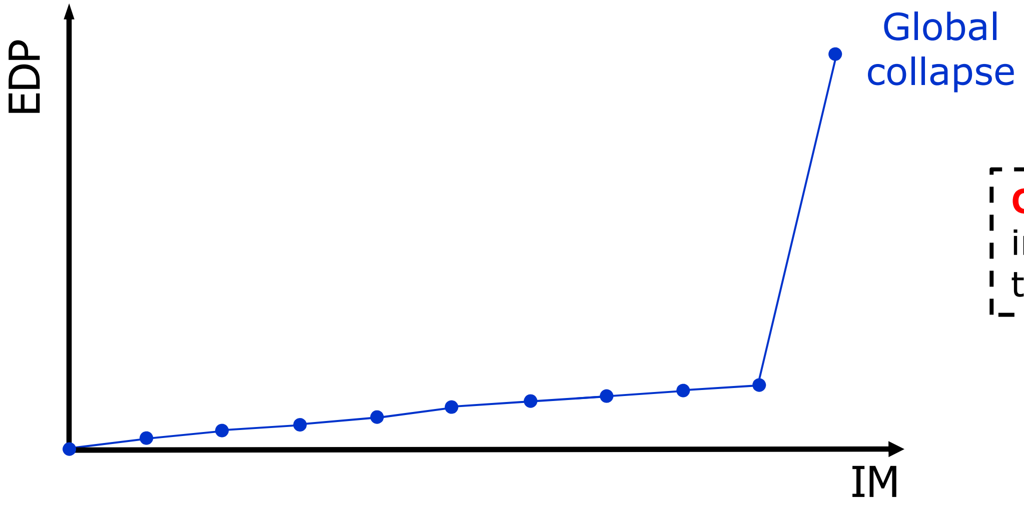
    } structural components
  - Floor acceleration: non-structural components
  - Interstory drift: structural & non-structural components
- **Peak values** of the above EDPs

Structural Analysis

- Separate treatment of global collapse since its probability does not change from a damageable group to the other

Methods of global collapse determination

Method I: Scaling a set of GMs for each intensity level



Global collapse: Unrealistic increase of EDP corresponding to a small increase in IM

Probability of global collapse for an intensity level:

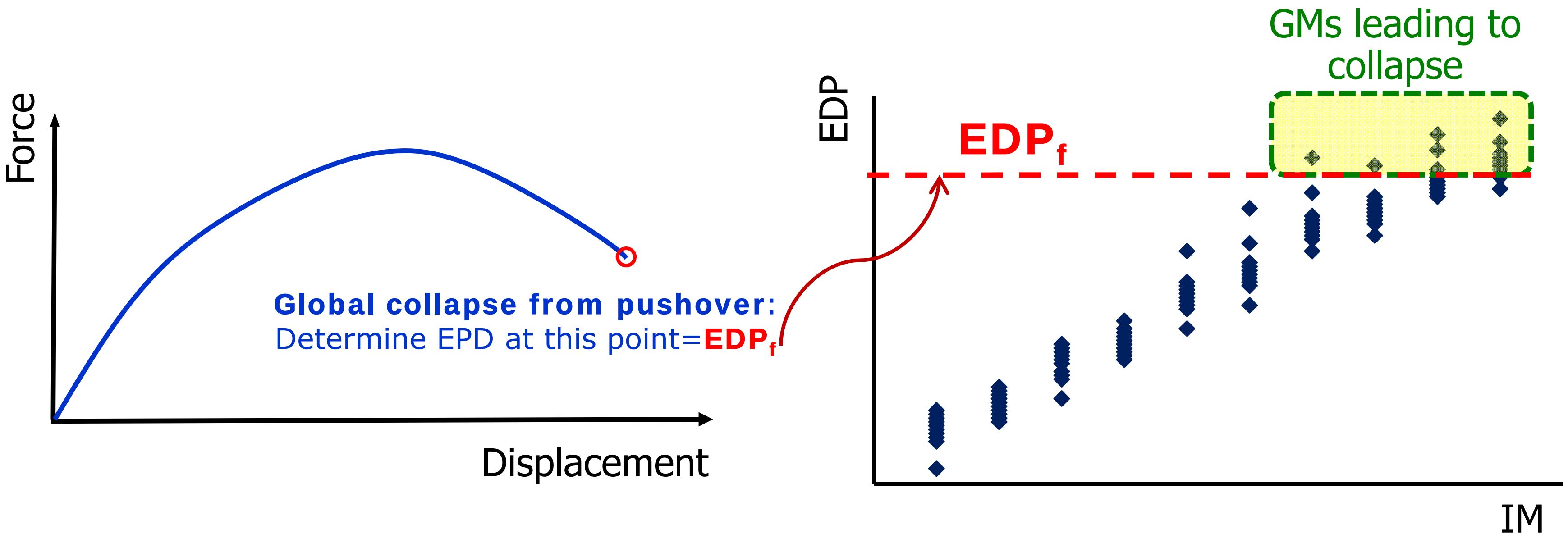
p(C|IM) = # of GMs leading to collapse / total # of GMs

Structural Analysis

- ❖ Separate treatment of global collapse since its probability does not change from a damageable component to the other.

Methods of global collapse determination

Method II: Use of unscaled GMs



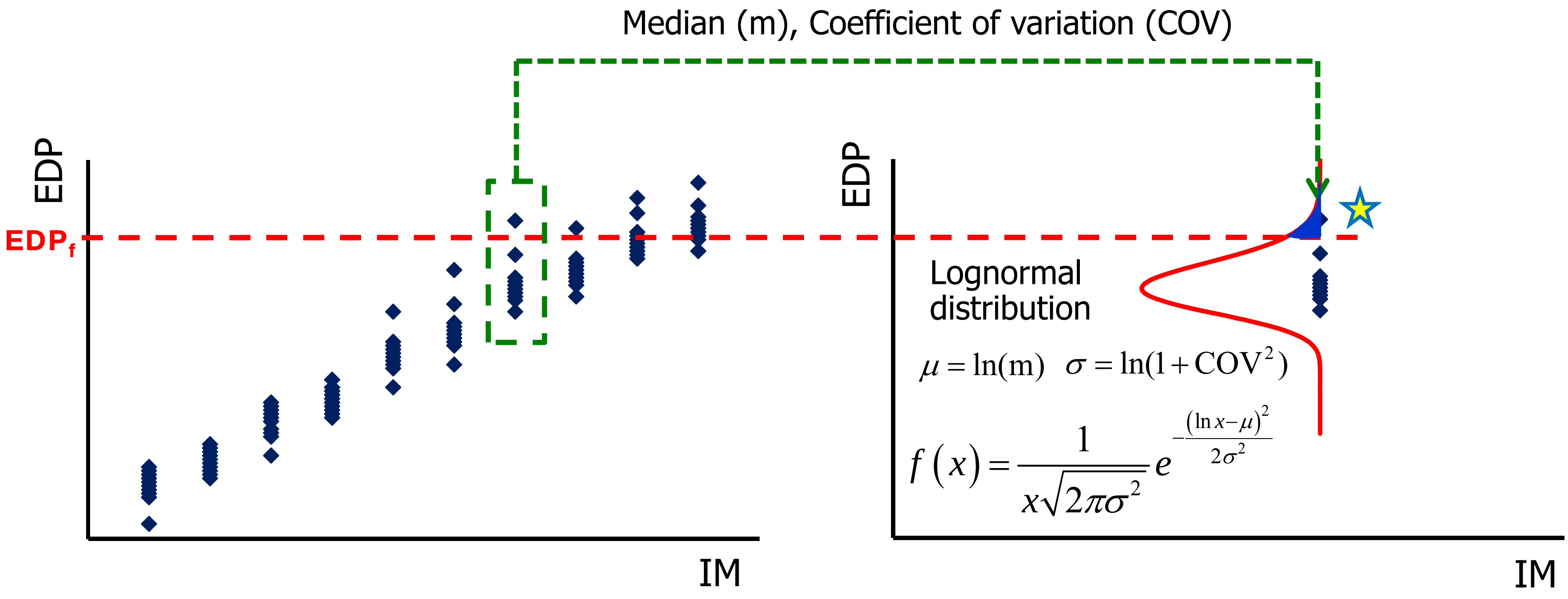
GMs leading to collapse

Global collapse from pushover:
Determine EPD at this point=**EDP_f**

Structural Analysis

Methods of global collapse determination

Method II: Use of unscaled GMs



$$\mu = \ln(\mathrm{m}) \quad \sigma = \ln(1+\mathrm{COV}^2)$$

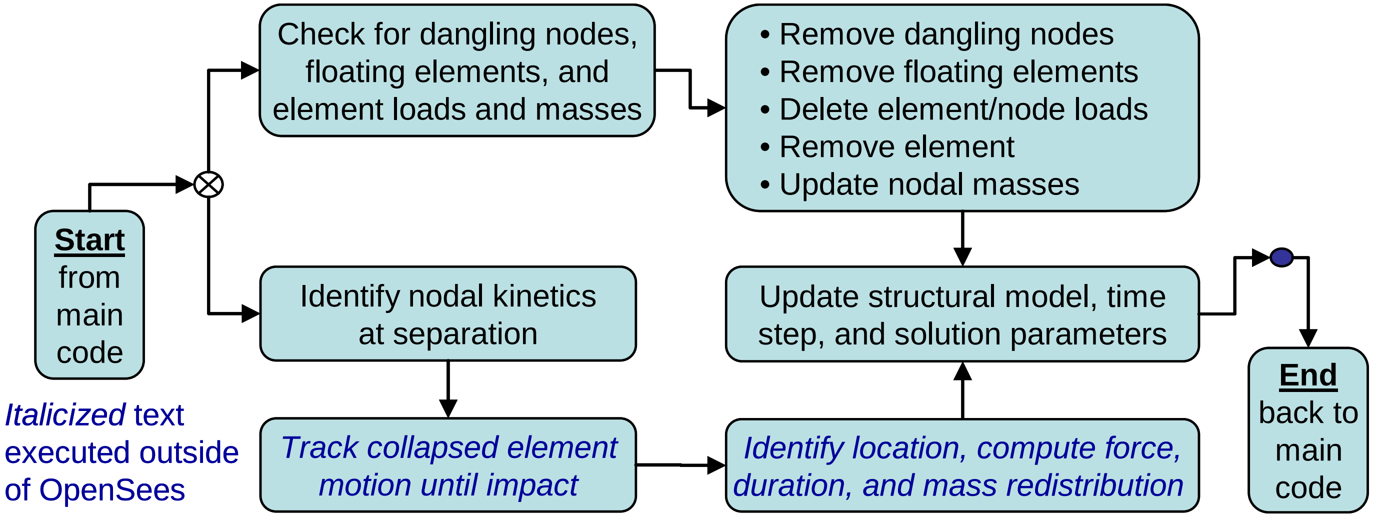
$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$$

a) p(C|IM) = # of GMs leading to collapse / total # of GMs

b) p(C|IM) = shaded area ☆

Structural Analysis

Progressive Collapse: A realistic representation of collapse in OpenSees



Start from main code

Check for dangling nodes, floating elements, and element loads and masses

- Remove dangling nodes
- Remove floating elements
- Delete element/node loads
- Remove element
- Update nodal masses

Update structural model, time step, and solution parameters

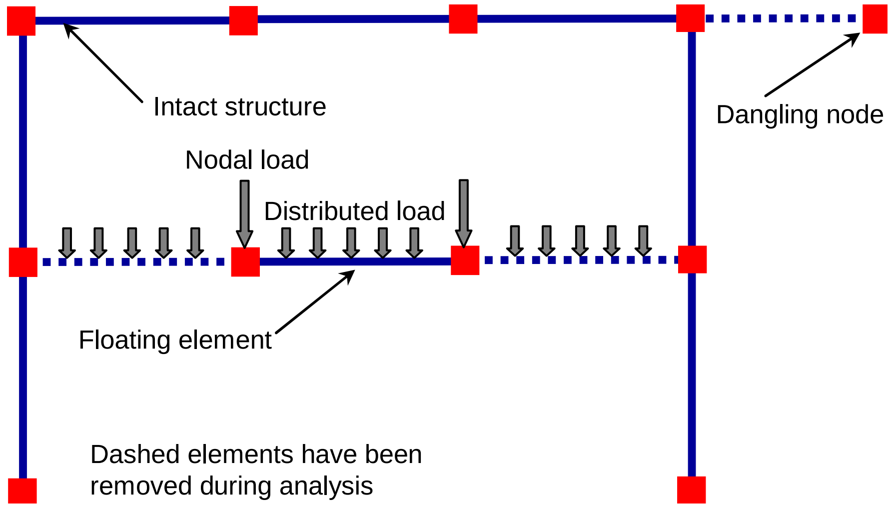
End back to main code

Identify nodal kinetics at separation

Track collapsed element motion until impact

Identify location, compute force, duration, and mass redistribution

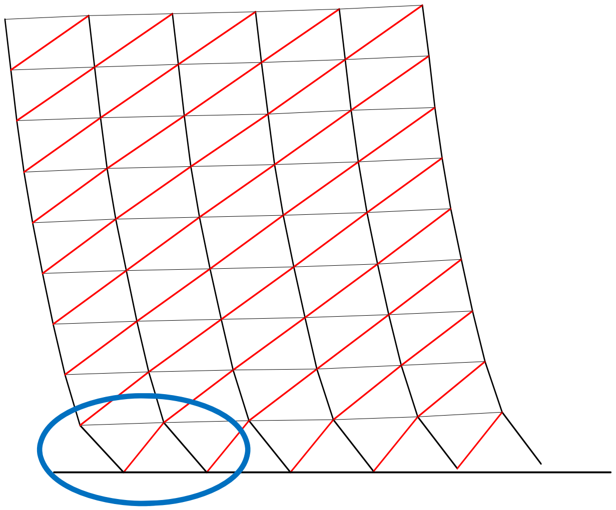
Italicized text executed outside of OpenSees



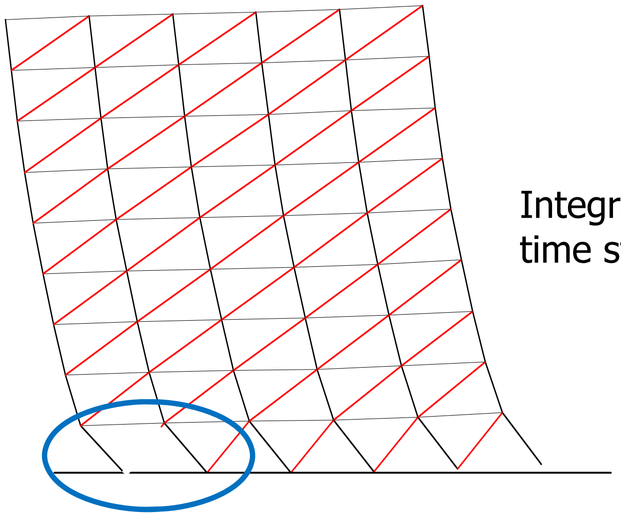
Talaat & Mosalam (2008)

Structural Analysis

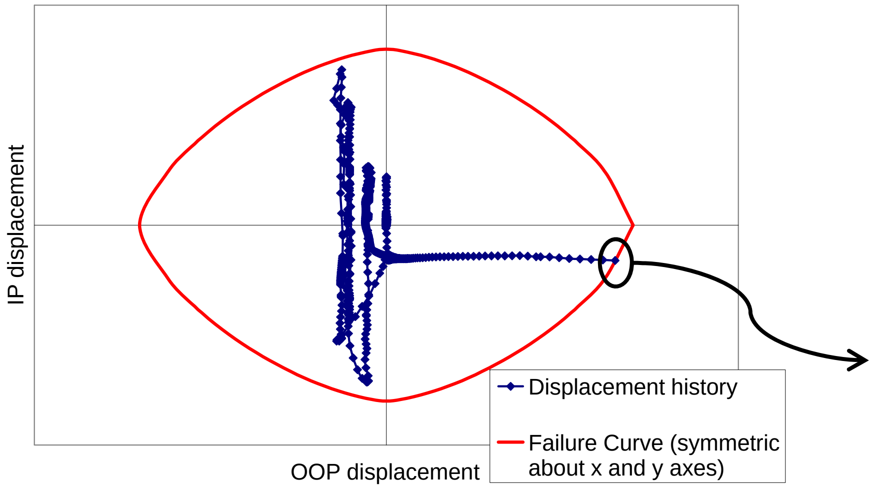
Progressive Collapse: A realistic representation of collapse in OpenSees



Integration
time step i-1



Integration
time step i

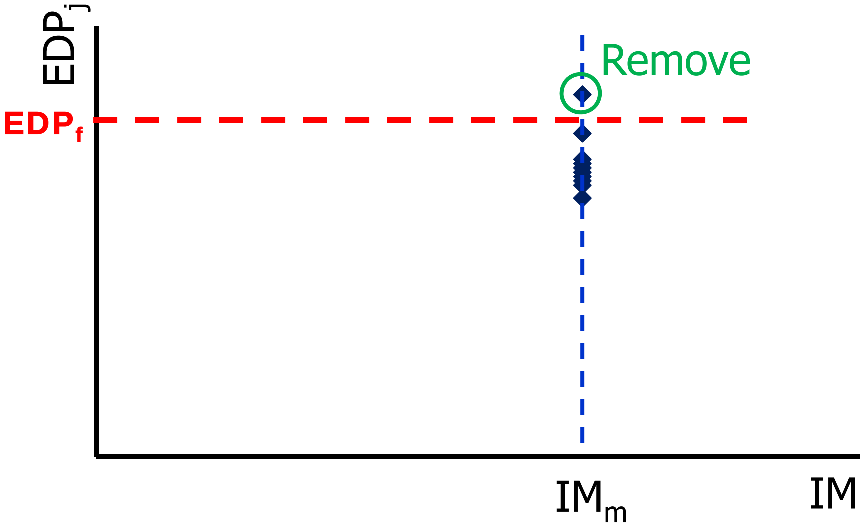


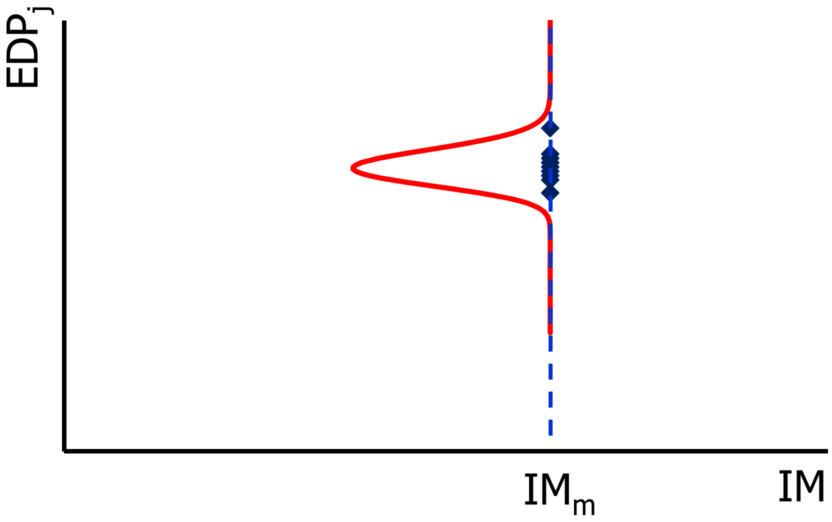
@Integration
time step i

http://opensees.berkeley.edu/wiki/index.php/Infill_Wall_Model_and_Element_Removal

Structural Analysis

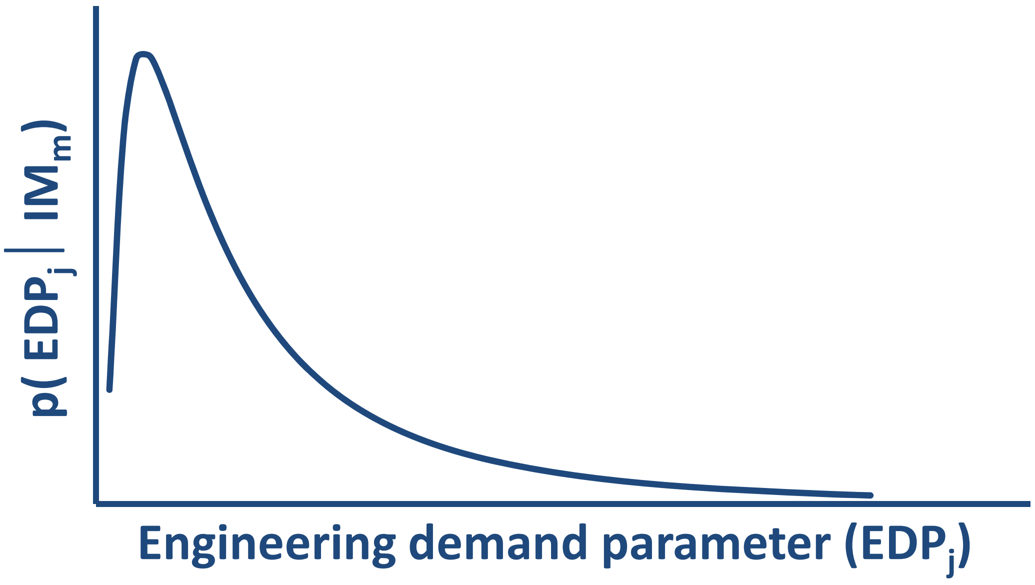
Outcome of Structural Analysis: Probability of each value (index i) of each EDP (index j) for each hazard level (index m): $\mathbf{p(EDP_j^i | Im_m)}$





Structural Analysis

Outcome of Structural Analysis: Probability of each value (index i) of each EDP (index j) for each hazard level (index m): $\mathbf{p(EDP_j^i \,|\, Im_m)}$



Damage Analysis

- **PEER PBEE objective**: Performance definition in terms of the direct interest of not only engineers, but also various stakeholders
- **Damage analysis**: Third analysis stage to achieve this objective
- **Damage analysis objective**: Estimate physical damage (i.e. Damage Measure, **DM**) at the component or system levels as functions of the structural response
- **DMs**: Typically defined in terms of damage levels corresponding to repair measures needed to restore components of a facility to the original conditions (other definitions are possible)
- **DM definition example**: Repair with epoxy injections (**light**); Repair with jacketing (**moderate**); Element replacement (**severe or collapse**)

Damage Analysis

- **Differences in path of achieving the same EDP & uncertainty in capacity**: A specific value of EDP corresponds to various DMs with different probabilities ← Uncertainty in damage analysis

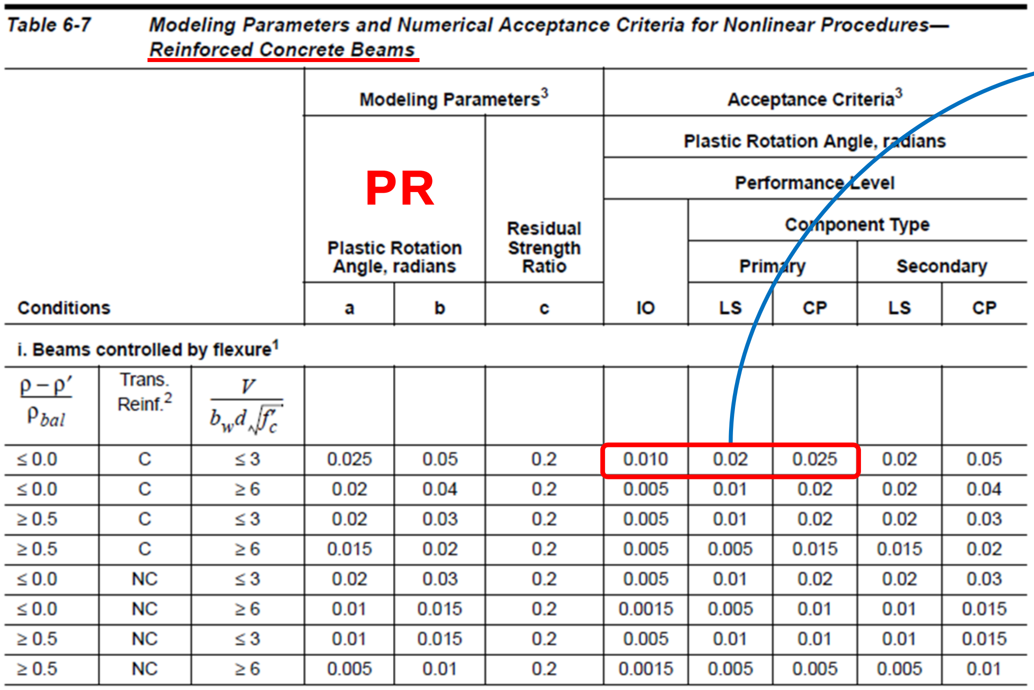


Table 6-7 *Modeling Parameters and Numerical Acceptance Criteria for Nonlinear Procedures— Reinforced Concrete Beams*

			Modeling Parameters[3]			Acceptance Criteria[3]				
						Plastic Rotation Angle, radians				
						Performance Level				
			PR Plastic Rotation Angle, radians		Residual Strength Ratio		Component Type			
							Primary		Secondary	
Conditions			a	b	c	IO	LS	CP	LS	CP
i. Beams controlled by flexure[1]										
$\frac{\rho-\rho'}{\rho_{bal}}$	Trans. Reinf.[2]	$\frac{V}{b_w d\sqrt{f'_c}}$								
≤ 0.0	C	≤ 3	0.025	0.05	0.2	0.010	0.02	0.025	0.02	0.05
≤ 0.0	C	≥ 6	0.02	0.04	0.2	0.005	0.01	0.02	0.02	0.04
≥ 0.5	C	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.03
≥ 0.5	C	≥ 6	0.015	0.02	0.2	0.005	0.005	0.015	0.015	0.02
≤ 0.0	NC	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.03
≤ 0.0	NC	≥ 6	0.01	0.015	0.2	0.0015	0.005	0.01	0.01	0.015
≥ 0.5	NC	≤ 3	0.01	0.015	0.2	0.005	0.01	0.01	0.01	0.015
≥ 0.5	NC	≥ 6	0.005	0.01	0.2	0.0015	0.005	0.005	0.005	0.01

FEMA-356

- If PR<0.01 → DM = IO
- If 0.01<PR<0.02 → DM = LS
- If 0.02<PR<0.025 → DM = CP

Examples:

- PR = 0.005 → DM = IO with p=100%
- PR = 0.015 → DM = LS with p=100%
- PR = 0.022 → DM = CP with p=100%
- PR = 0.030 → DM = Collapse with p=100%

Damage Analysis

FEMA-356

- PR = 0.005 → DM = IO with p=100%
- PR = 0.015 → DM = LS with p=100%
- PR = 0.022 → DM = CP with p=100%
- PR = 0.030 → DM = Collapse with p=100%

Note: Probability values are chosen arbitrarily for PEER-PBEE

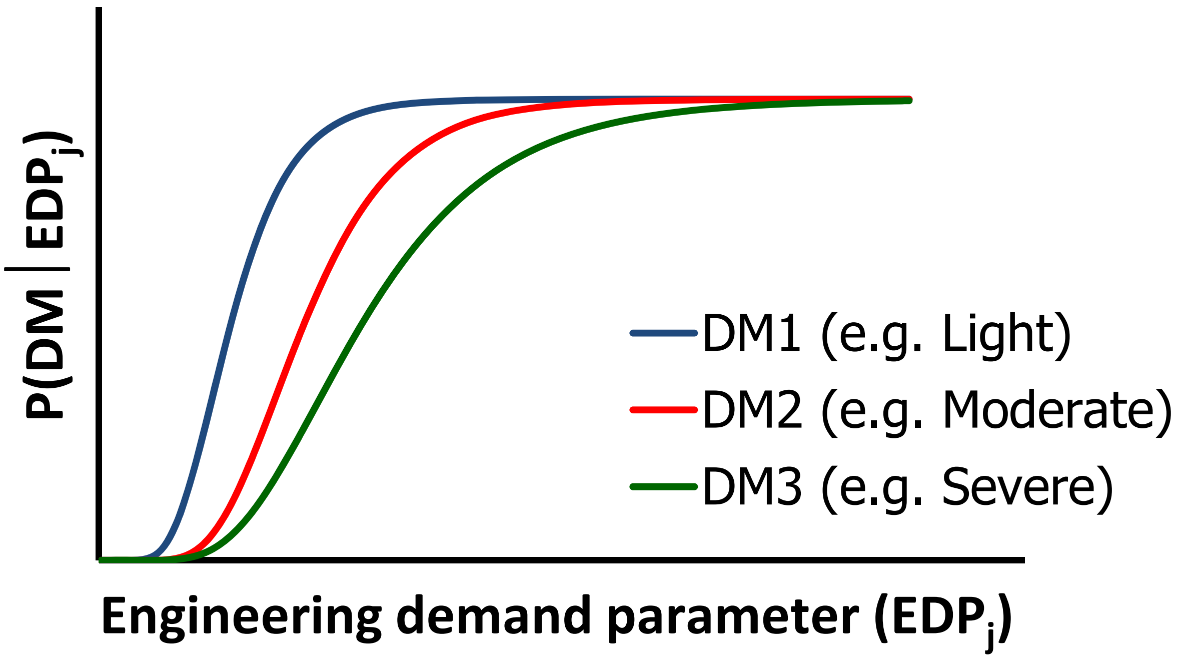
PEER-PBEE

- PR = 0.005 → DM = IO with p=70%, DM = LS with p=20%,
  DM = CP with p=18%, DM= collapse with p=2%
- PR = 0.015 → DM = IO with p=15%, DM = LS with p=60%,
  DM = CP with p=20%, DM= collapse with p=5%
- PR = 0.022 → DM = IO with p=5%, DM = LS with p=15%,
  DM = CP with p=60%, DM= collapse with p=20%
- PR = 0.030 → DM = IO with p=2%, DM = LS with p=12%,
  DM = CP with p=21%, DM= collapse with p=65%

Damage Analysis

- **Tool used in damage analysis:**

  Fragility function: POE of a DM for different values of an EDP



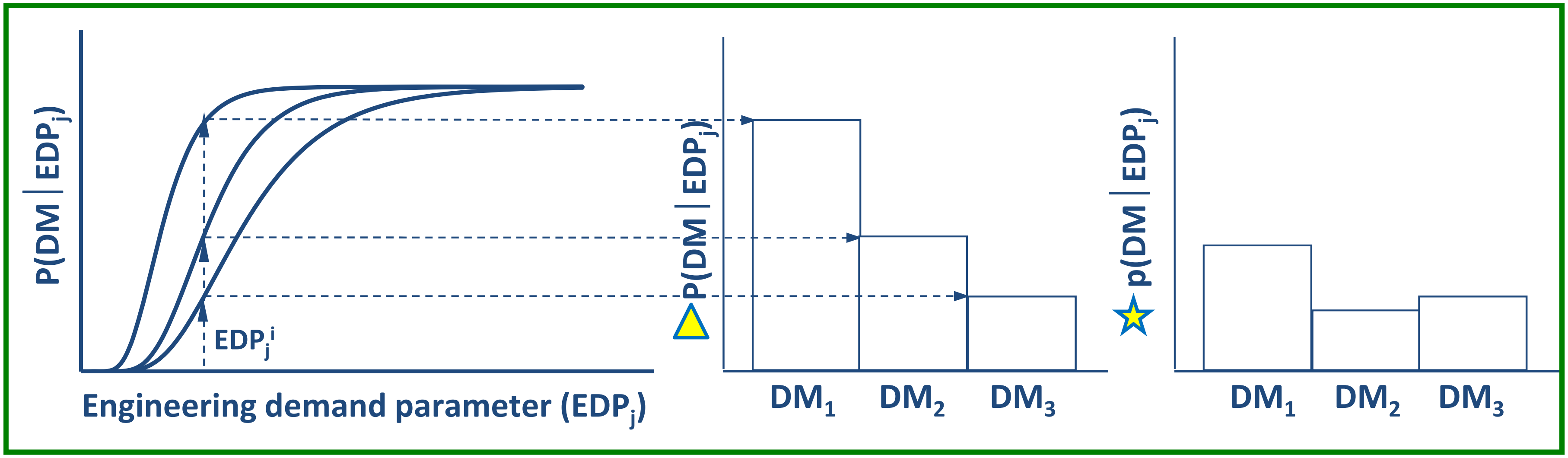
Damage Analysis

- **Fragility function determination**:
  - Analytical simulations
  - Experimental simulations (Hybrid simulation or shake table tests)
  - Generic functions based on expert opinion (not preferred)

- Damageable parts of a structure are divided into damageable groups:
  - Each damageable group consists of components that are affected by the same EDP in a similar way
  - The components in a group have the same fragility functions
  - **Example**: Bohl (2009) used **16** different groups for a steel moment frame building including: (1) the structural system, (2) the exterior enclosure, (3) drift-sensitive and (4) acceleration-sensitive non-structural elements, and (5) office content for each floor

Damage Analysis

Outcome of Damage Analysis: Probability of each DM value (index k) for each value (index i) of each EDP (index j): **$p(DM_k|EDP^i_j)$**



$$\text{for } k = 1 : \#\text{ of DM levels}$$

$$p(DM_k|EDP_j^i) = P(DM_k|EDP_j^i) \quad \text{if } k = \#\text{ of DM levels}$$

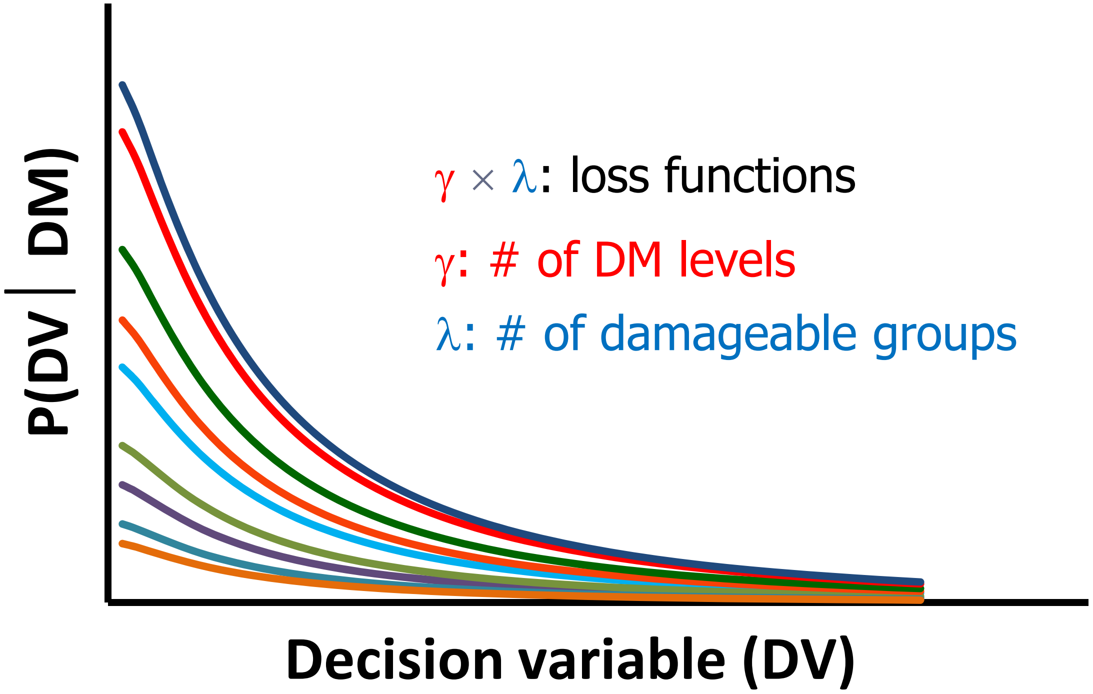
$$\underset{☆}{p(DM_k|EDP_j^i)} = \underset{△}{P(DM_k|EDP_j^i)} - P(DM_{k+1}|EDP_j^i) \quad \text{otherwise}$$

Loss Analysis

- Last (Fourth) analysis stage in PEER PBEE Formulation
- **Damage information obtained from damage analysis**: Converted to the final decision variables (DVs)
- **Commonly utilized DVs**:
  - Fatalities
  - Economic loss
  - Repair duration
  - Injuries
- **Distribution of damage within the damageable group**: A specific value of DM corresponds to various DVs with different probabilities ← Uncertainty in loss analysis
- **Economic loss or repair cost as DV**: Uncertainty originating from the economical values, e.g. fluctuation in the market prices, is included

Loss Analysis

- **Tool used in loss analysis:**
  Loss function: POE of a DV for different damageable groups and DMs



$\gamma \times \lambda$: loss functions

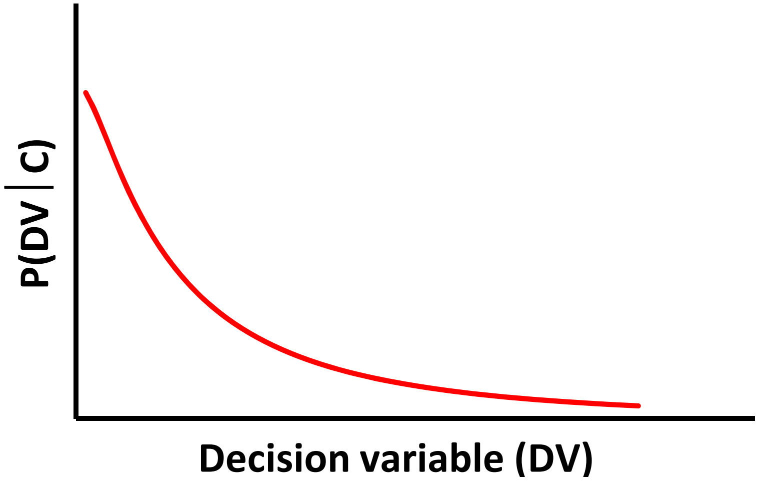
γ: # of DM levels

λ: # of damageable groups

Loss Analysis

Loss function for collapse:

- Krawinkler (2005) assumed a lognormal distribution for P(DV|C)
- The expected value can be assumed as the total cost of the structural and nonstructural components of the facility
- **Following factors can be considered as sources of variance**:
  - ❖ Lack of information about all the present structural and non-structural components
  - ❖ Lack of monetary value information about the components
  - ❖ Fluctuation in market prices



Combination of Analyses

Total probability theorem:

Given n mutually exclusive events* $A_1,\ldots,A_n$ whose probabilities sum to 1.0, then the probability of an arbitrary event B:

$$p(B) = p(B|A_1)\,p(A_1) + p(B|A_2)\,p(A_2) + \ldots + p(B|A_n)\,p(A_n)$$

$$p(B) = \sum_i p(B|A_i)\;p(A_i)$$

Conditional probability of B given the presence of A_i

Probability of A_i

*Occurrence of any one of them automatically implies the non-occurrence of the remaining n−1 events

Combination of Analyses

PEER PBEE combination of analyses: based on total probability theorem

Hazard Analysis

End product: $P(DV^n)=\sum_m P(DV^n|IM_m)\,p(IM_m)$

POE of the n^{th} value of the DV of the facility

Structural Analysis: Probability of no-collapse & of collapse

$$P(DV^n|IM_m)=P(DV^n|NC,IM_m)\,p(NC|IM_m)+P(DV^n|C)\,p(C|IM_m)$$

Loss Analysis: Loss function for collapse

$$P(DV^n|NC,IM_m)=\sum_j P(DV_j^n|NC,IM_m)$$

Structural Analysis

$$P(DV_j^n|NC,IM_m)=\sum_i P(DV_j^n|EDP_j^i)\,p(EDP_j^i|IM_m)$$

$$P(DV_j^n|EDP_j^i)=\sum_k P(DV_j^n|DM_k)\,p(DM_k|EDP_j^i)$$

Loss Analysis

Damage Analysis

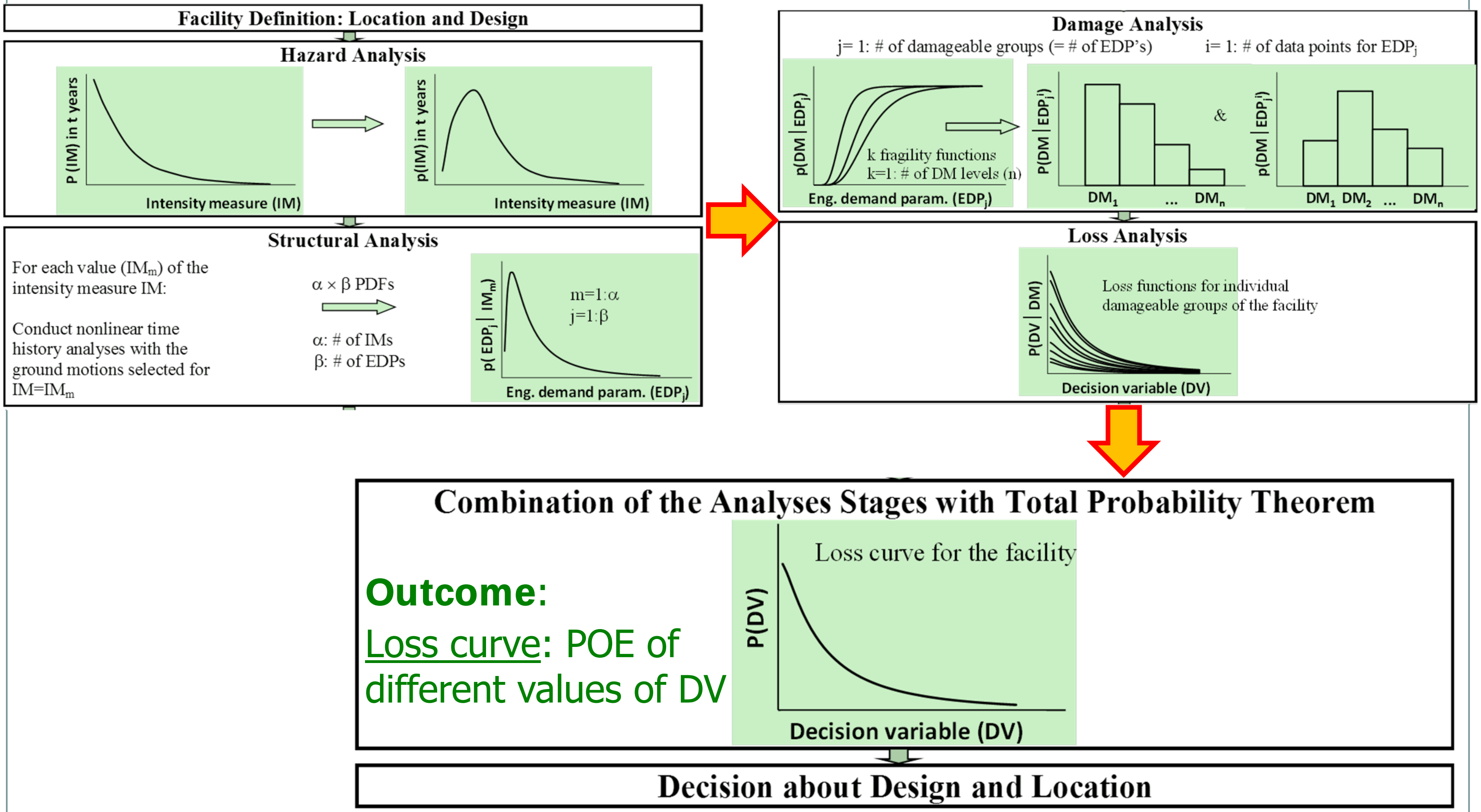
m: index for IM

j: index for damageable groups (DG)

i: index for EDP

k: index for DM

Combination of Analyses



Facility Definition: Location and Design

Hazard Analysis

Structural Analysis

For each value (IM_m) of the intensity measure IM:

Conduct nonlinear time history analyses with the ground motions selected for $IM=IM_m$

α × β PDFs

α: # of IMs

β: # of EDPs

Damage Analysis

j= 1: # of damageable groups (= # of EDP's)   i= 1: # of data points for EDP_j

Loss Analysis

Combination of the Analyses Stages with Total Probability Theorem

Outcome:

Loss curve: POE of different values of DV

Decision about Design and Location

Combination of Analyses

Remark: *Loss*, *damage*, & *structural* analyses results are summed in a straightforward manner. However, integration of the *hazard* analysis into the formulation does not take place in such a way because of the presence of damageable groups and collapse and non-collapse cases.

Straightforward equation in case of a single DG and no collapse:

$$P\left(DV^{n}\right)=\sum_{m}\sum_{i}\sum_{k}\underbrace{P\left(DV^{n}\middle|DM_{k}\right)}_{\text{Loss}}\underbrace{p\left(DM_{k}\middle|EDP^{i}\right)}_{\text{Damage}}\underbrace{p\left(EDP^{i}\middle|IM_{m}\right)}_{\text{Structural}}\underbrace{p\left(IM_{m}\right)}_{\text{Hazard}}$$

Direct resemblance to the PEER PBEE framework equation:

$$\lambda(DV)=\int\int\int G\langle DV\,|\,DM\rangle\; dG\langle DM\,|\,EDP\rangle\; dG\langle EDP\,|\,IM\rangle\; d\lambda(IM)$$

λ: Mean Annual Frequency (MAF), G: Conditional probability

Combination of Analyses

Remark: POE of the DV in case of collapse, P(DV|C), is not conditioned on the IM, whereas the POE of the DV in case of no collapse, $P(DV|NC,IM_m)$, is conditioned on the IM because:

- **No collapse** case consists of different damage states and the contribution of each of these damage states to this case changes for different IMs. This is not the situation for **collapse** case.
- For example, loss function for slight damage has the highest contribution for a small value of IM, whereas the loss function for severe damage has the highest contribution for a large value of IM.

Combination of Analyses

Variation in the formulation: Replace POE (**P**) with expected value (**E**)

Hazard Analysis

End product: $E(DV^n) = \sum_m E(DV^n|IM_m)\, p(IM_m)$

E of the nth value of the DV of the facility

Structural Analysis: Probability of no-collapse & of collapse

$$E(DV^n|IM_m) = E(DV^n|NC, IM_m)\, p(NC|IM_m) + E(DV^n|C)\, p(C|IM_m)$$

Loss Analysis: Loss function for collapse

$$E(DV^n|NC, IM_m) = \sum_j E(DV_j^n|NC, IM_m)$$

Structural Analysis

$$E(DV_j^n|NC, IM_m) = \sum_i E(DV_j^n|EDP_j^i)\, p(EDP_j^i|IM_m)$$

$$E(DV_j^n|EDP_j^i) = \sum_k E(DV_j^n|DM_k)\, p(DM_k|EDP_j^i)$$

Loss Analysis **Damage Analysis**

m: index for IM

j: index for damageable groups (DG)

i: index for EDP

k: index for DM

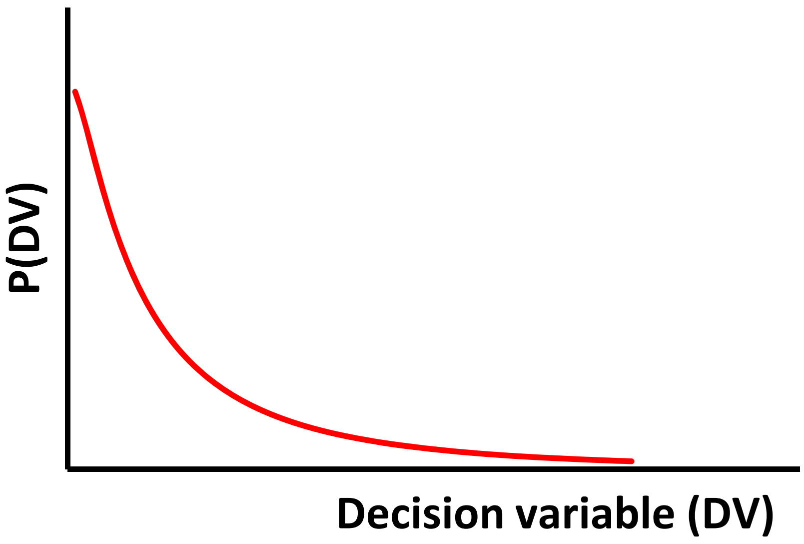
Combination of Analyses

Variation in the formulation: Replace POE (**P**) with expected value (**E**)

Outcome: Expected value of the decision variable

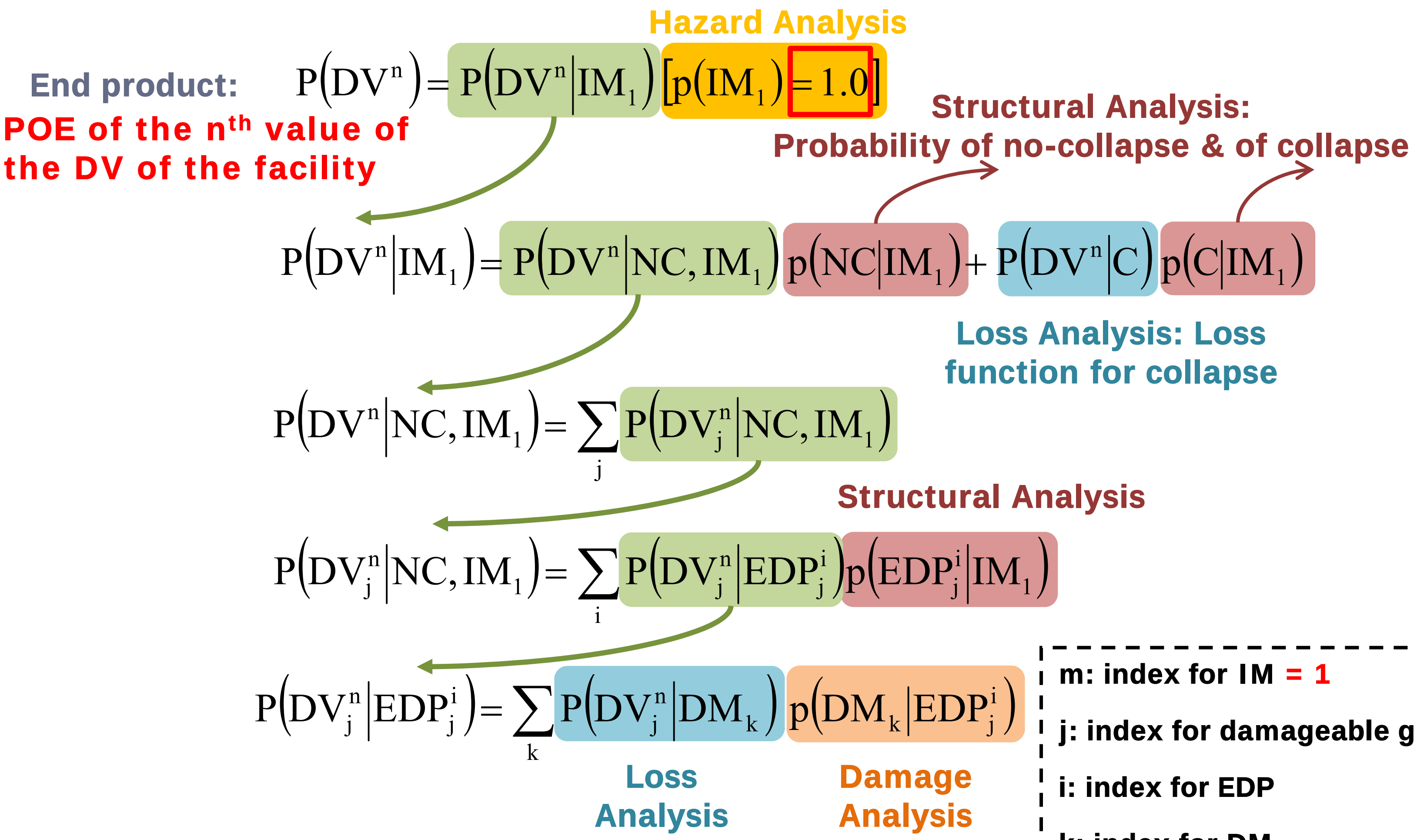
Instead of

Loss curve:



Combination of Analyses

Variation in the formulation: Consider a **single IM** value, **IM_1**



Hazard Analysis

End product:

$$P\left(DV^n\right) = P\left(DV^n \middle| IM_1\right)\left[p\left(IM_1\right) \boxed{= 1.0}\right]$$

POE of the nth value of the DV of the facility

Structural Analysis: Probability of no-collapse & of collapse

$$P\left(DV^n \middle| IM_1\right) = P\left(DV^n \middle| NC, IM_1\right) p\left(NC \middle| IM_1\right) + P\left(DV^n \middle| C\right) p\left(C \middle| IM_1\right)$$

Loss Analysis: Loss function for collapse

$$P\left(DV^n \middle| NC, IM_1\right) = \sum_j P\left(DV_j^n \middle| NC, IM_1\right)$$

Structural Analysis

$$P\left(DV_j^n \middle| NC, IM_1\right) = \sum_i P\left(DV_j^n \middle| EDP_j^i\right) p\left(EDP_j^i \middle| IM_1\right)$$

$$P\left(DV_j^n \middle| EDP_j^i\right) = \sum_k P\left(DV_j^n \middle| DM_k\right) p\left(DM_k \middle| EDP_j^i\right)$$

Loss Analysis

Damage Analysis

m: index for IM = 1

j: index for damageable groups (DG)

i: index for EDP

k: index for DM

Application Options

How can an engineer use PEER PBEE method?

1. Evaluation of a traditional code-based design in a performance-based probabilistic approach. This application is appropriate in the current state of traditional code-based design if the engineer wants to introduce performance-based enhancements to the mandatory code-based design.
2. Evaluation of the performance of an existing structure or the outcome of different retrofit interventions.
3. Use of the methodology directly as a design tool, e.g. for decision-making amongst different design alternatives. This type of application is expected to gain widespread use when the probabilistic **PBED** methods start to be employed as a standard design method.

Thank you