FEM STATIC AND DYNAMIC MODELING OF ROMAN VAULTED MONUMENTS IN *OPUS CAEMENTICIUM*

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STRUCTURAL FORMS

The combination of geometrical shapes and structural materials designed to transmit forces (functional loads, weights, etc.) in buildings.

STRUCTURAL KNOWLEDGE

Structural engineering depends upon a detailed knowledge of forces, mechanics, and materials to understand and predict how structures support and resist self-weight and imposed loads.

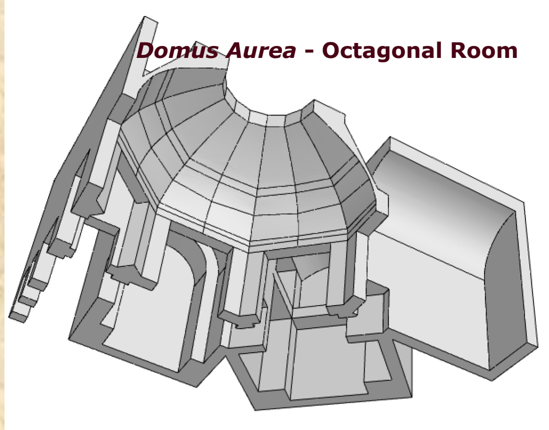
"The concept of failure is central to the design process, and it is by thinking in terms of obviating failure that successful designs are achieved." H. Petroski. *Design Paradigms*.

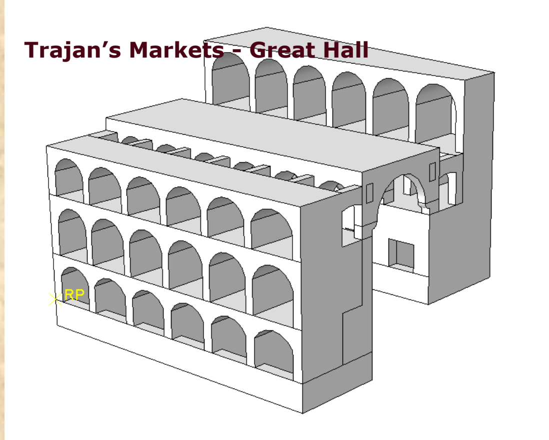
HISTORICAL DEVELOPMENT

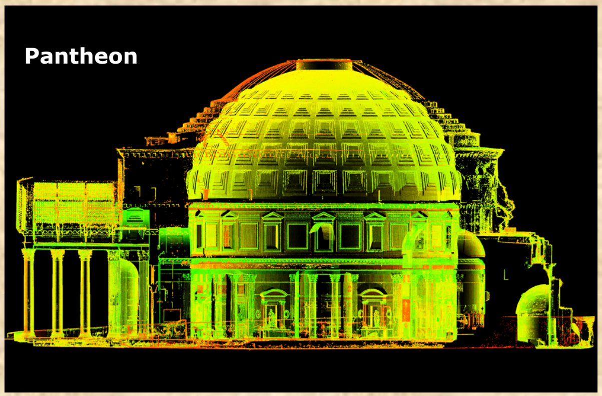
Roman engineers developed the *structural form* to levels of innovations unparalleled until the introduction of structural steel and reinforced concrete in the nineteenth century.

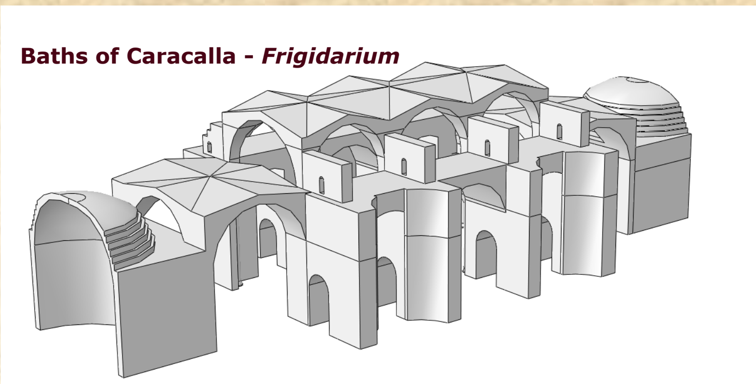
Imperial vaulted architecture

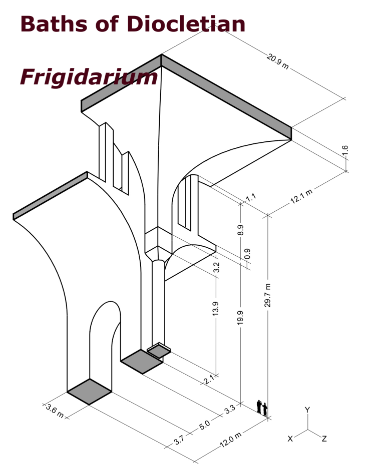
- Large to gigantic vaults and domes
- Complex 3D (solid) configurations
- Built with un-reinforced pozzolanic concrete (*opus caementicium*)
- Structural analysis requires advanced numerical modeling
- Finate Element method provides the analytical and computational framework





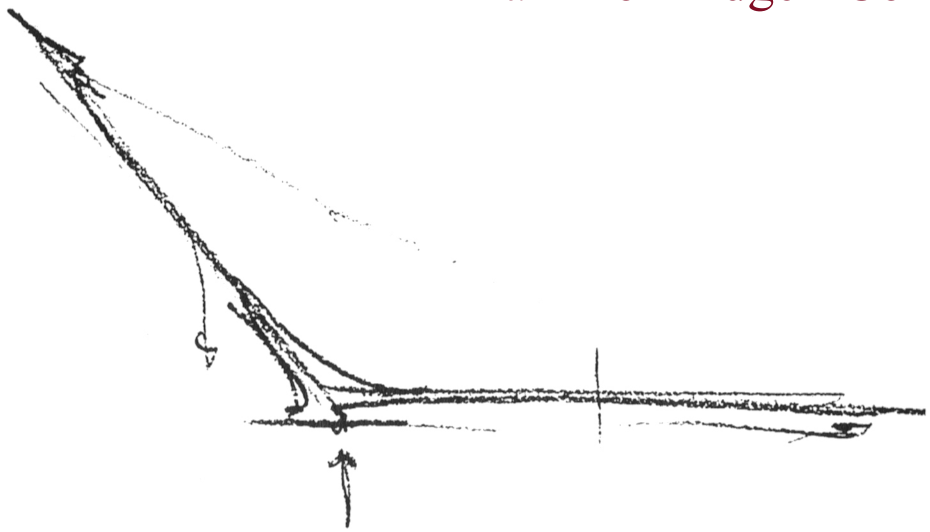


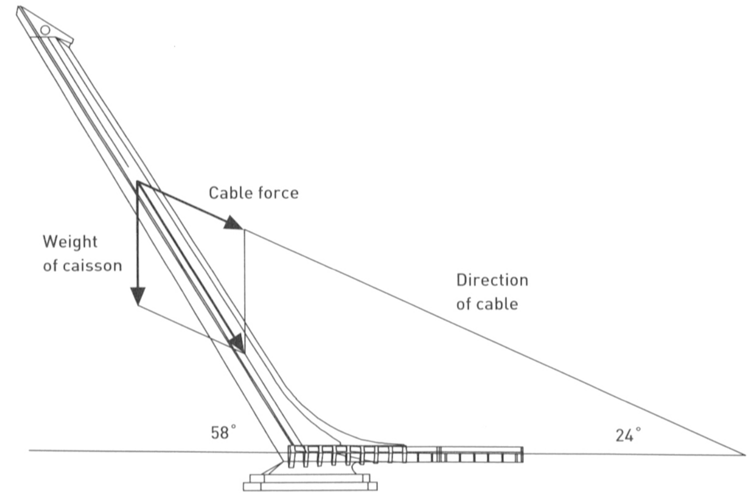


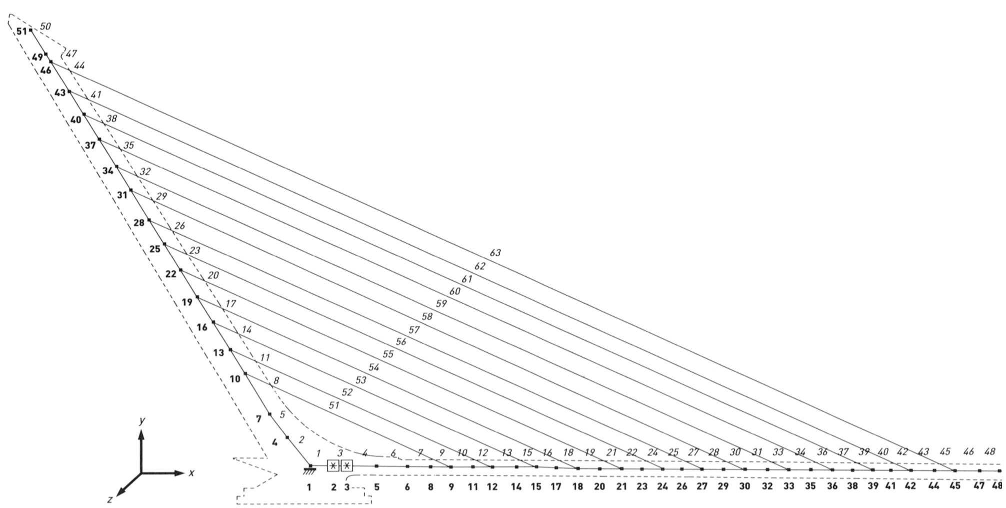


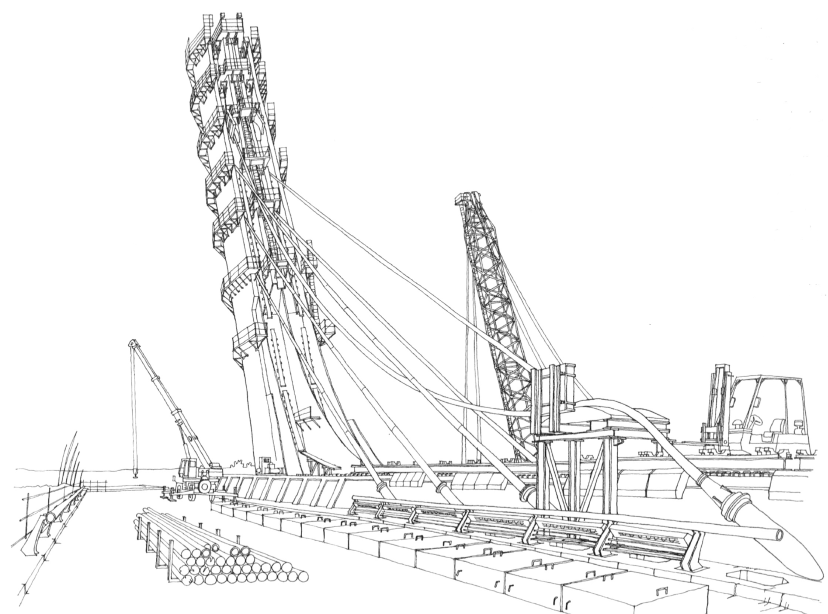
Seville - Alamillo Bridge - Santiago Calatrava - 1992
form: cable-stayed beam   material: steel cables and reinforced concrete

Alamillo Bridge - Concept to Construction

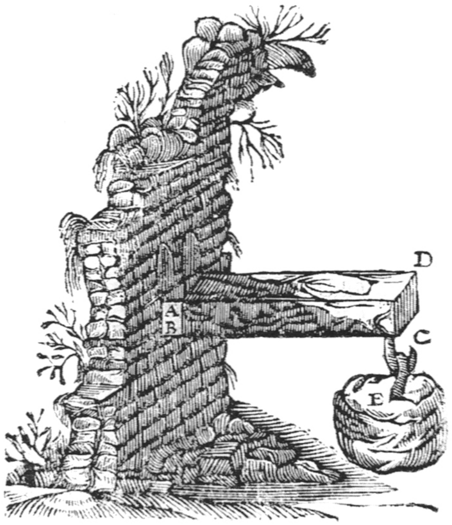




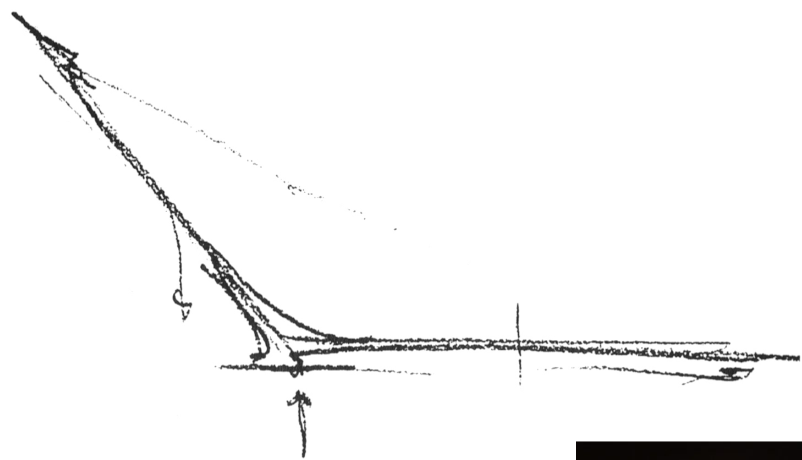




Alamillo Bridge - Lineage



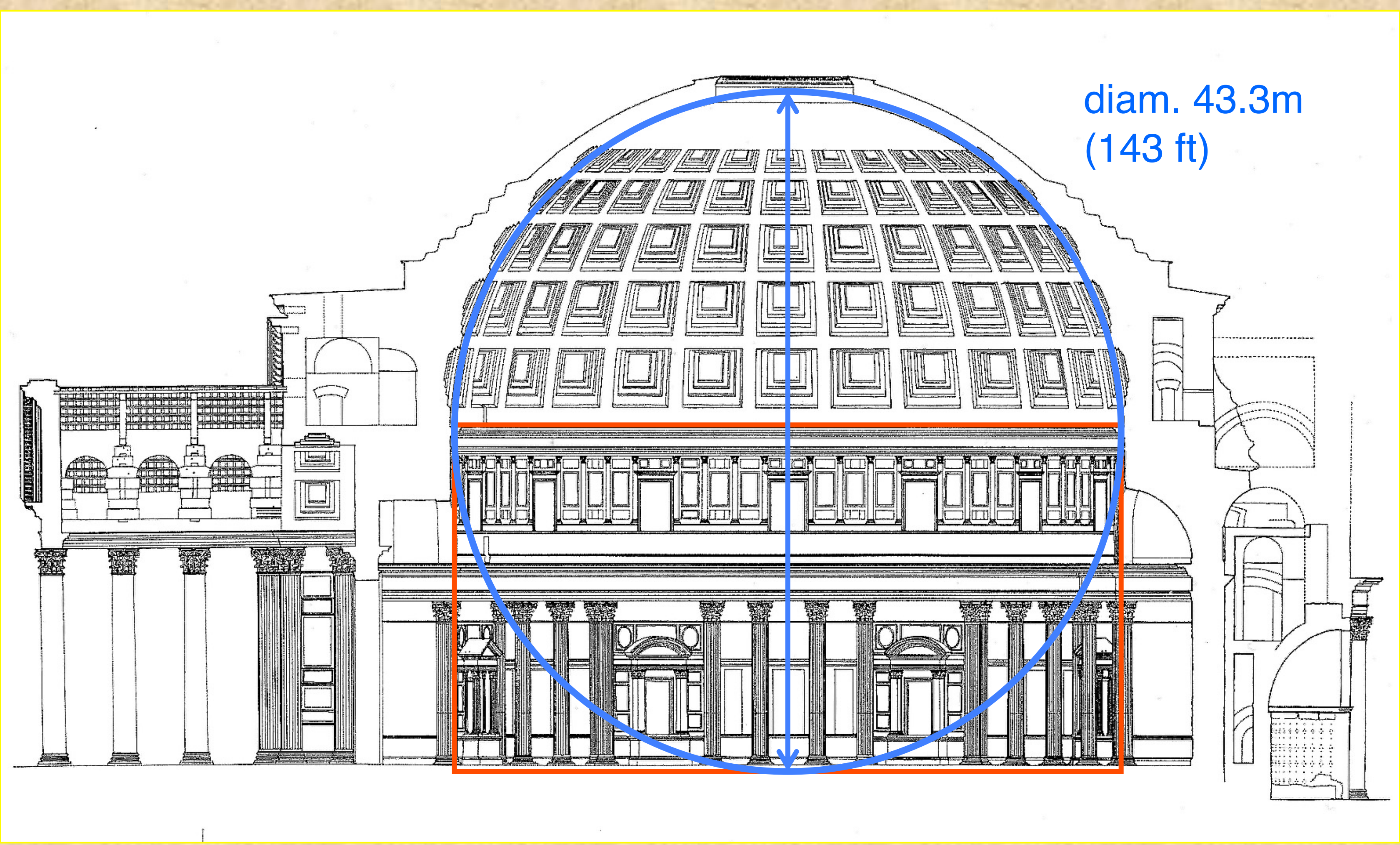
Galileo *Discorsi* 1638



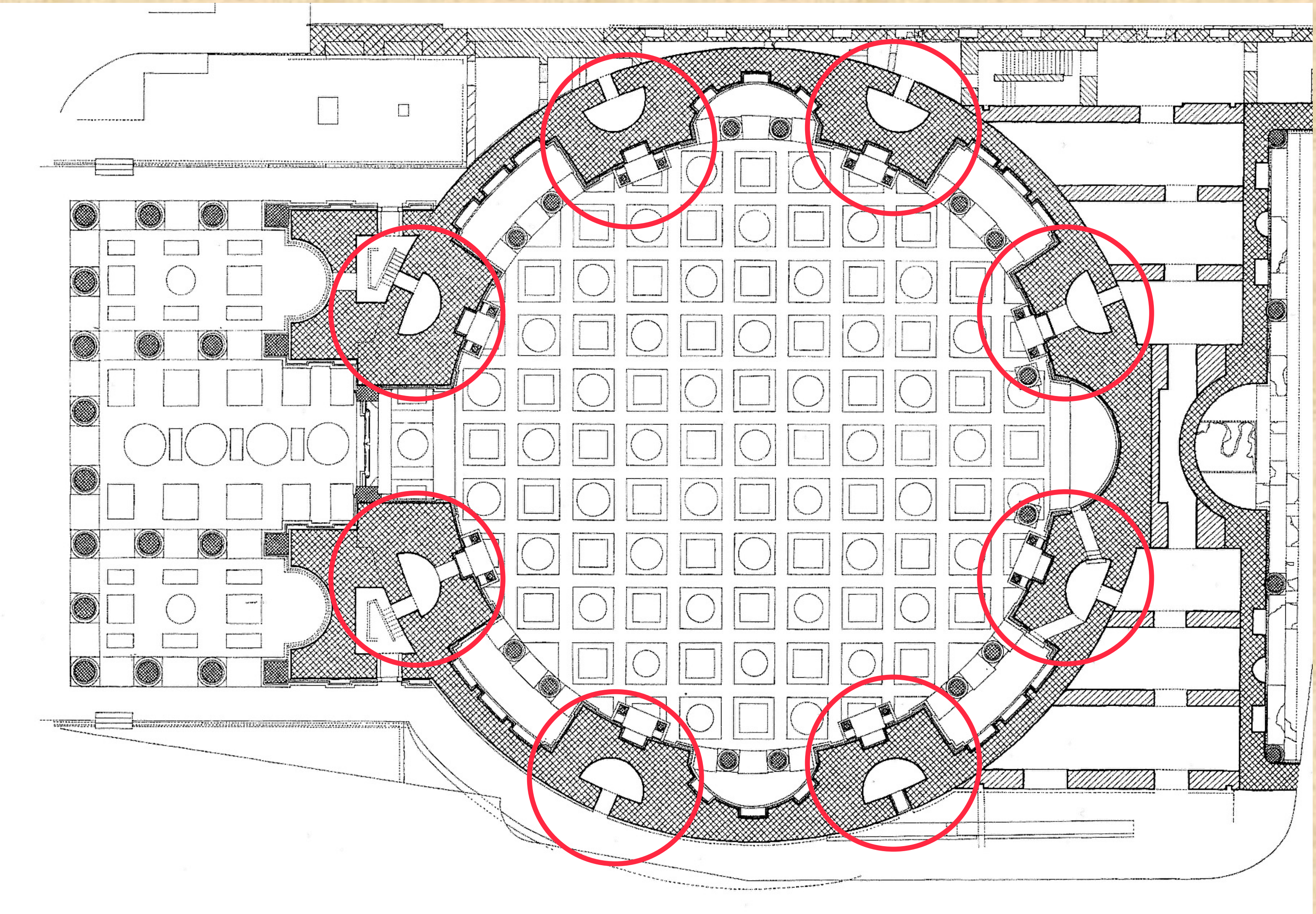
Rome - Pantheon (Hadrian) - AD 118-125
form: dome - material: pozzolanic concrete (*opus caementicium*)

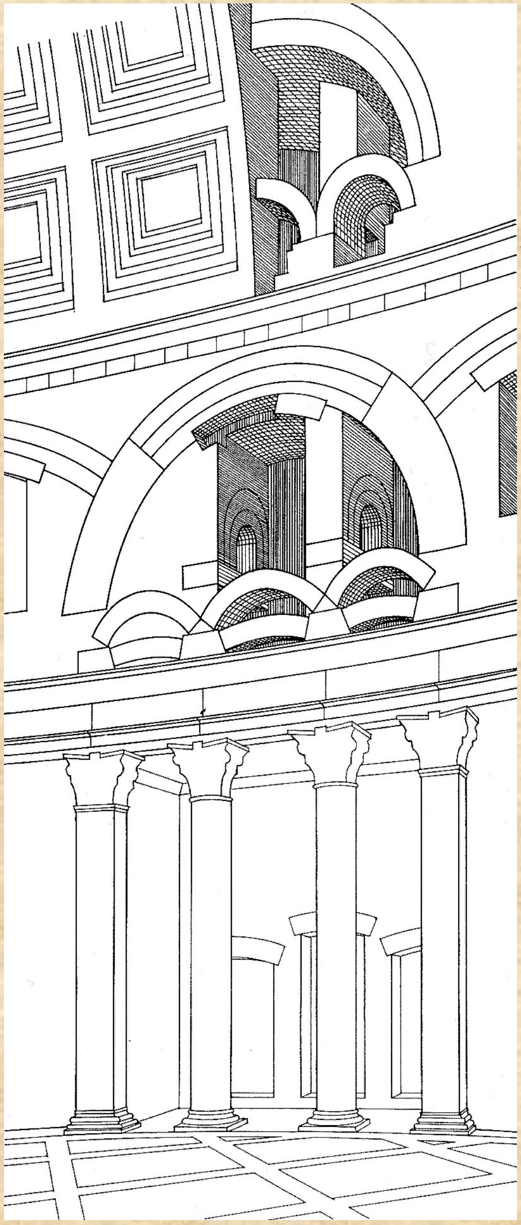
Rome - Pantheon - Interior and dome

Pantheon - cross section

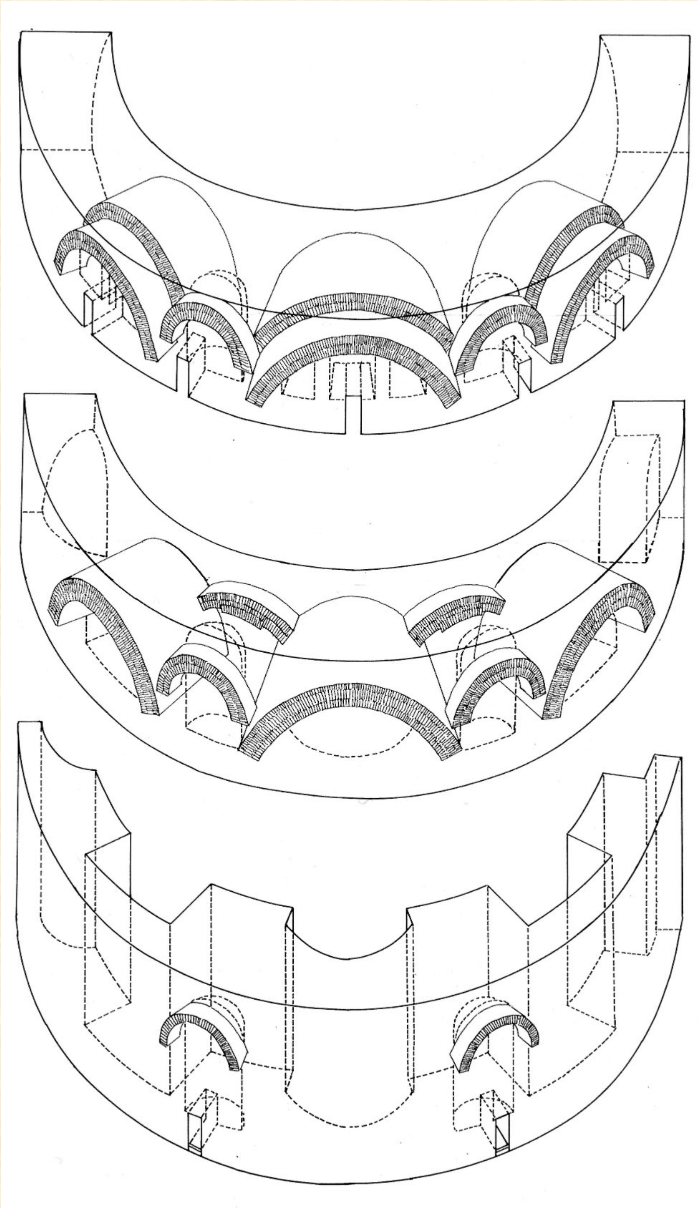


Pantheon - plan





Pantheon wall structure



Pantheon (AD 118-125) - Largest un-reinforced concrete (or masonry) dome ever built

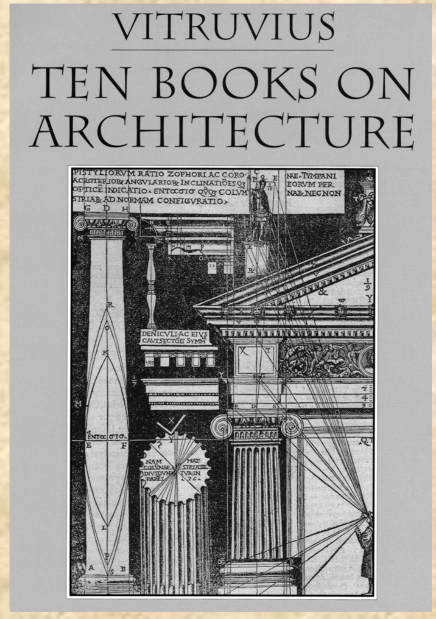
More structurally daring than Brunelleschi's dome (Florence Cathedral 1430) or Michelangelo's dome (Saint Peter - Rome 1564)

Dome is in excellent structural conditions (after nineteen centuries of service…)


Pantheon


Saint Peter

Modern structural codes would not allow the construction of the dome of the Pantheon due to the inherent structural weakness of the material (un-reinforced concrete)



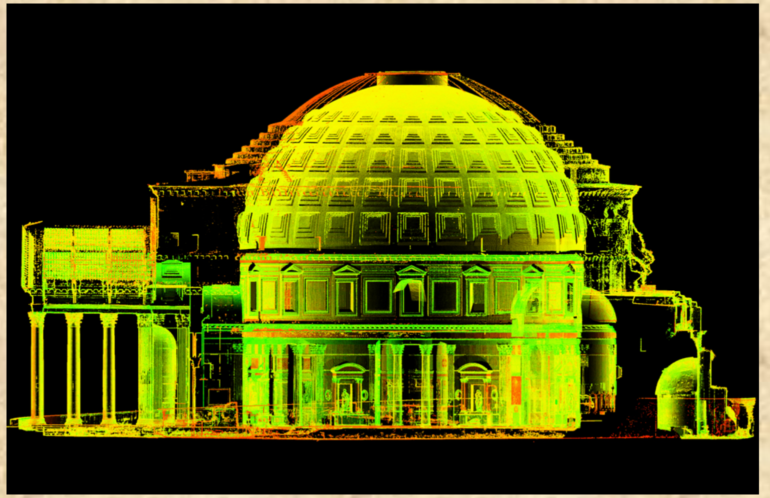
Vitruvius
De Architectura
30 - 20 BC

Rome - Pantheon

Who is the designer?
What is the structural "lineage"?

How was technical knowledge acquired?
How was it transmitted?

How was it built?
What is the mechanics of *opus caementicium*?



Outline

- *Opus caementicium*
- Trajan’s Markets: Great Hall
- Baths of Diocletian: Frigidarium
- Baths of Caracalla: Frigidarium
- Nero’s *Domus Aurea*
- Pantheon
- Future directions

Roman Concrete (*opus caementicium*)

Man-made conglomerate consisting primarily of

QUICKLIME + POZZOLAN + WATER + AGGREGATE

- Burning limestone (travertine) ==> QUICKLIME (*calx*)
- Volcanic ash ==> POZZOLAN (*pulvis puteolanus*)
- tuf, travertine, basalt, or brick fragments ==> AGGREGATE (*caementa*)

PREPARATION

Quicklime and pozzolan are mixed with water to form mortar (excellent cementing agent).

Layer of aggregate is placed over mortar in a wooden form.

Mortar is tamped into form.

Concrete hardens and form is removed.

No iron bar/grid reinforcements (with some exceptions)

Roman Concrete (*opus caementicium*)

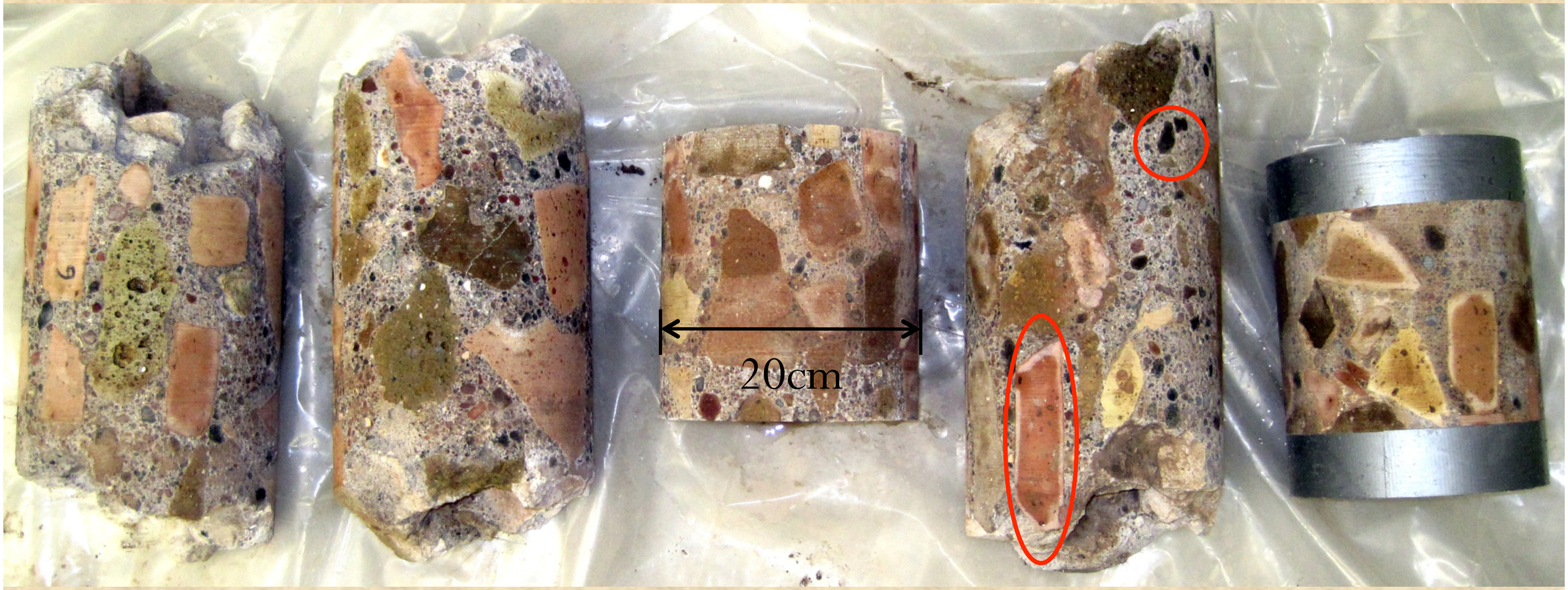
ADVANTAGES

- Good strength in compression.
- Lighter than stone or brick.
- Can be formed into complex 3D shapes (domes, vaults).
- Less expensive than stone or brick.
- Can be used under water (hydraulic cement).

DISADVANTAGES
Small strength in tension (but not zero).
Requires (long) curing time.
Must be protected from atmospheric agents (brick or tile facing).
Domes and vaults require complex and expensive frameworks.

OUTSTANDING STRUCTURAL MATERIAL

Great Hall -- *Opus Caementicium* (walls)



Multiple types, sizes, and morphologies of dm-scale coarse aggregate

Mortar made with multiple types of volcanic ash and ground tuff

Mechanical behavior of modern concrete*

Composite material consisting of different size aggregate particles embedded in a cement paste matrix (mortar)

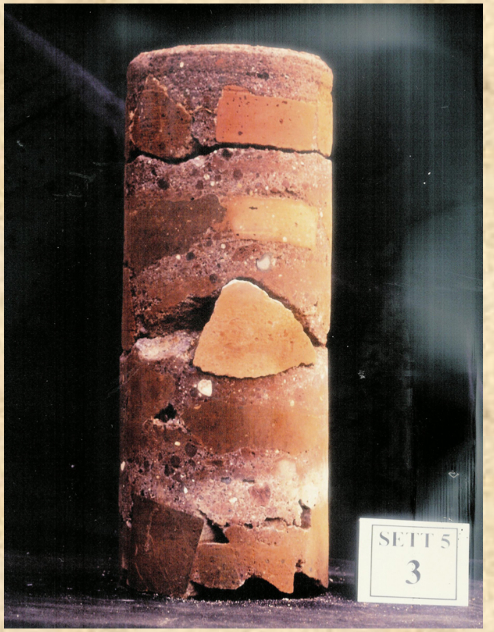
Excellent mechanical strength in compression but comparatively poor in tension

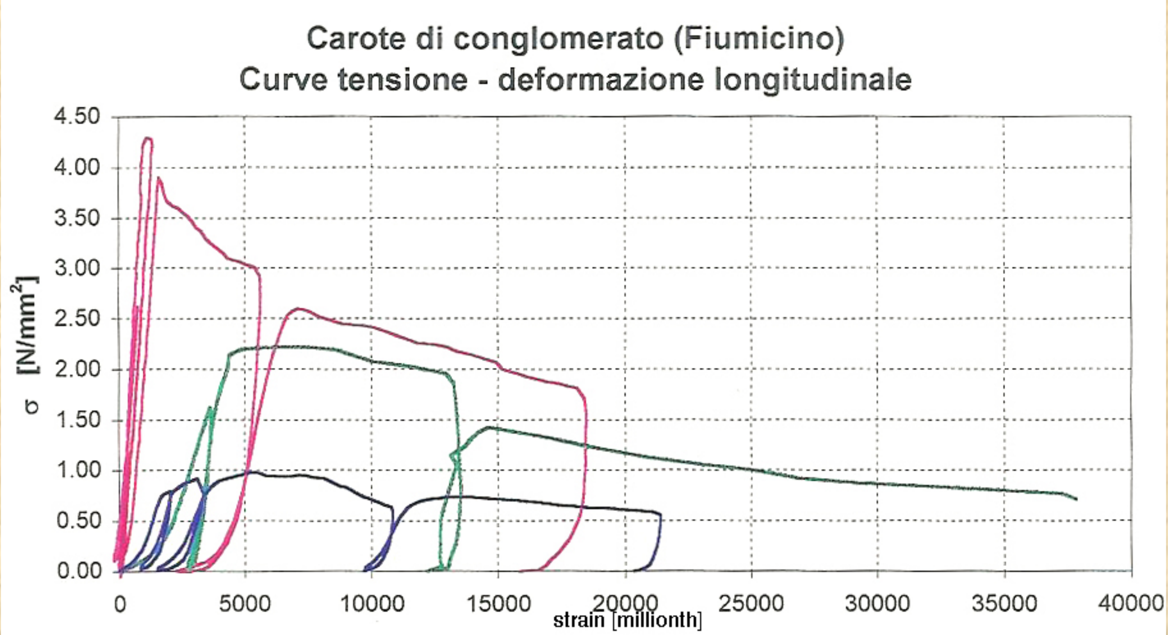
Aggregate-mortar interface constitutes the weakest link in the composite system and the primary cause for the low tensile strength

Mechanical testing of *opus caementicium* follows similar procedures to those used for testing modern concrete

*A. M. Neville, *Properties of Concrete*, 4th ed., New York: Wiley, 1996.
W.F.Chen and A.F. Saleeb, *Constitutive Equations for Engineering Materials,* New York: Wiley,1982.

Mechanical testing of *opus caementicium* - compression*





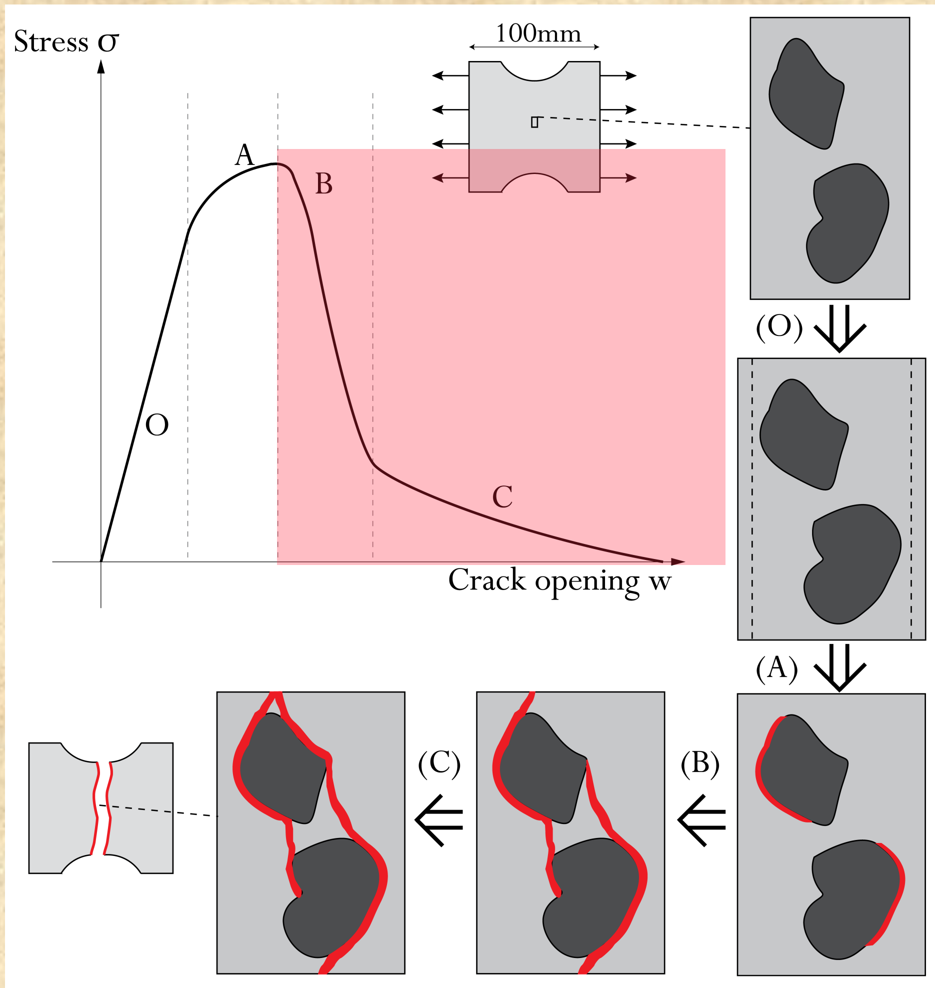
Specimens from Trajan’s port, Hadrian’s villa, and Basilica of Maxentius

Stress-strain curve similar to modern concrete (nonlinear response and hysteresis likely induced by fracture propagation)

Ultimate strength in compression: avg = 4.55 MPa, range = 1 - 6.7 MPa, generally correlated with quality of concrete confection, best quality found in highly loaded areas

*A. Samuelli Ferretti, “Materiali da costruzione e technologie costruttive del patrimonio archeologico”, Internal reports, Università degli Studi di Roma, La Sapienza, 1995. Also, several articles in “Materiali e Strutture. Problemi di Conservazione”, VII, 2-3, 1997

Mechanical testing of *opus caementicium* - traction

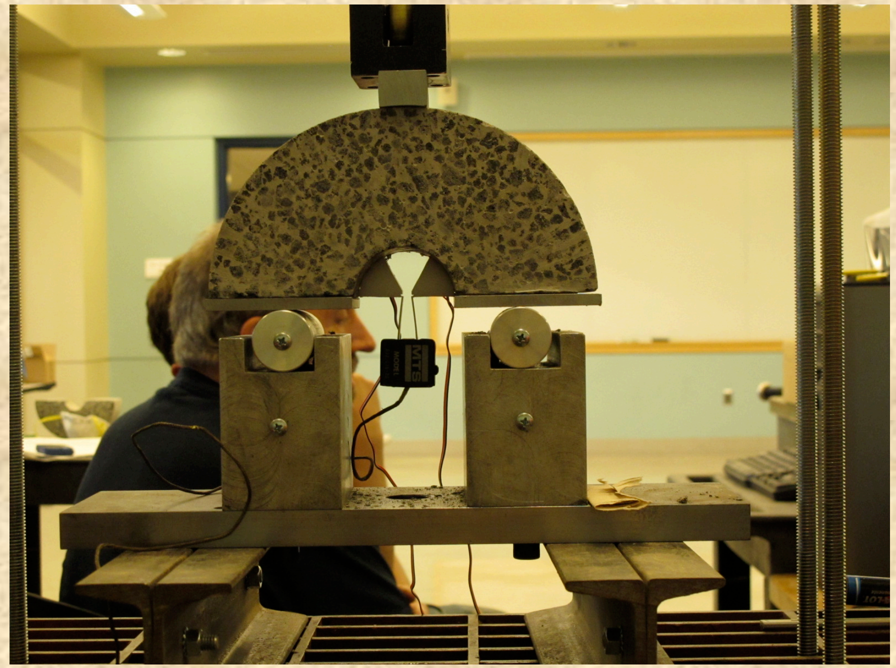


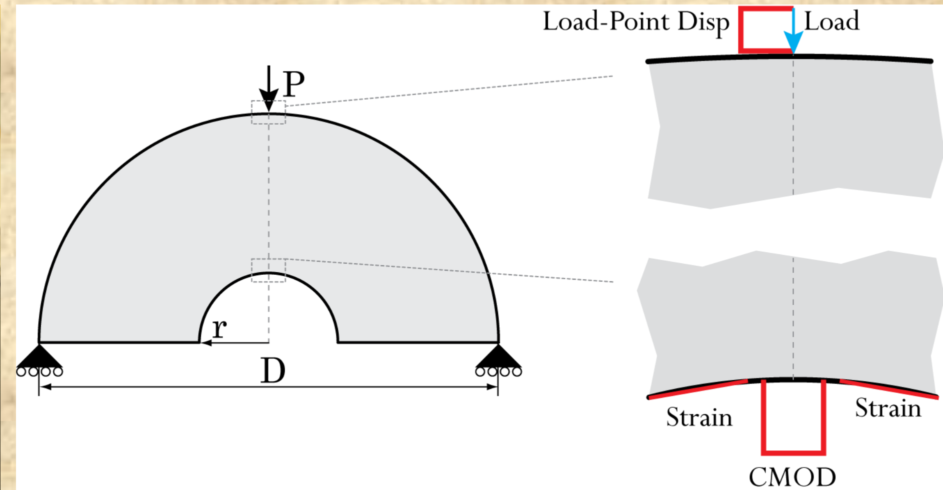
Cohesive Fracture model

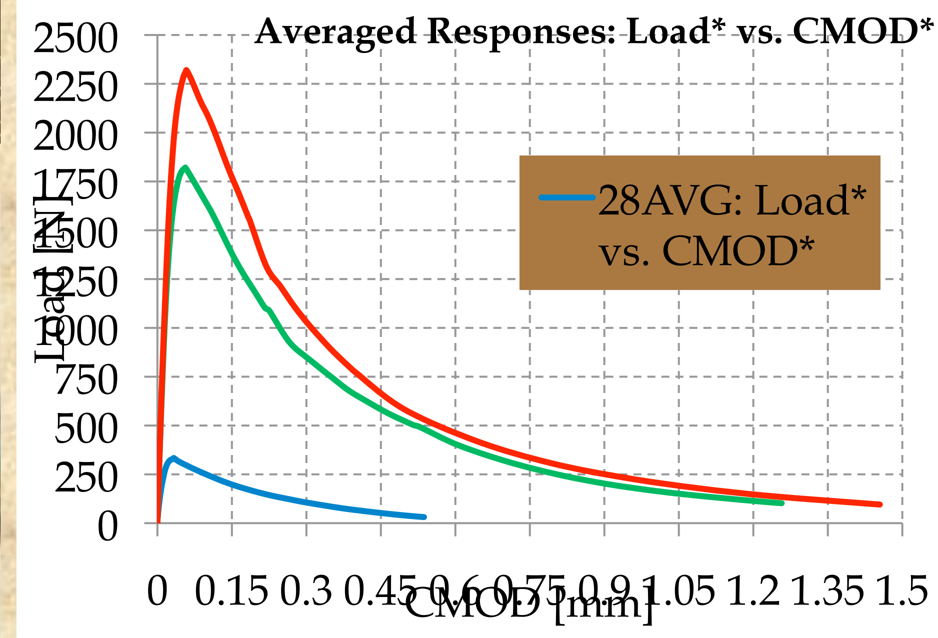
- (O) elastic response
- (A) stable micro-cracking
- (B) macro-crack localization and propagation
- (C) crack bridging

In this particular test specimen, crack bridging leads to complete failure

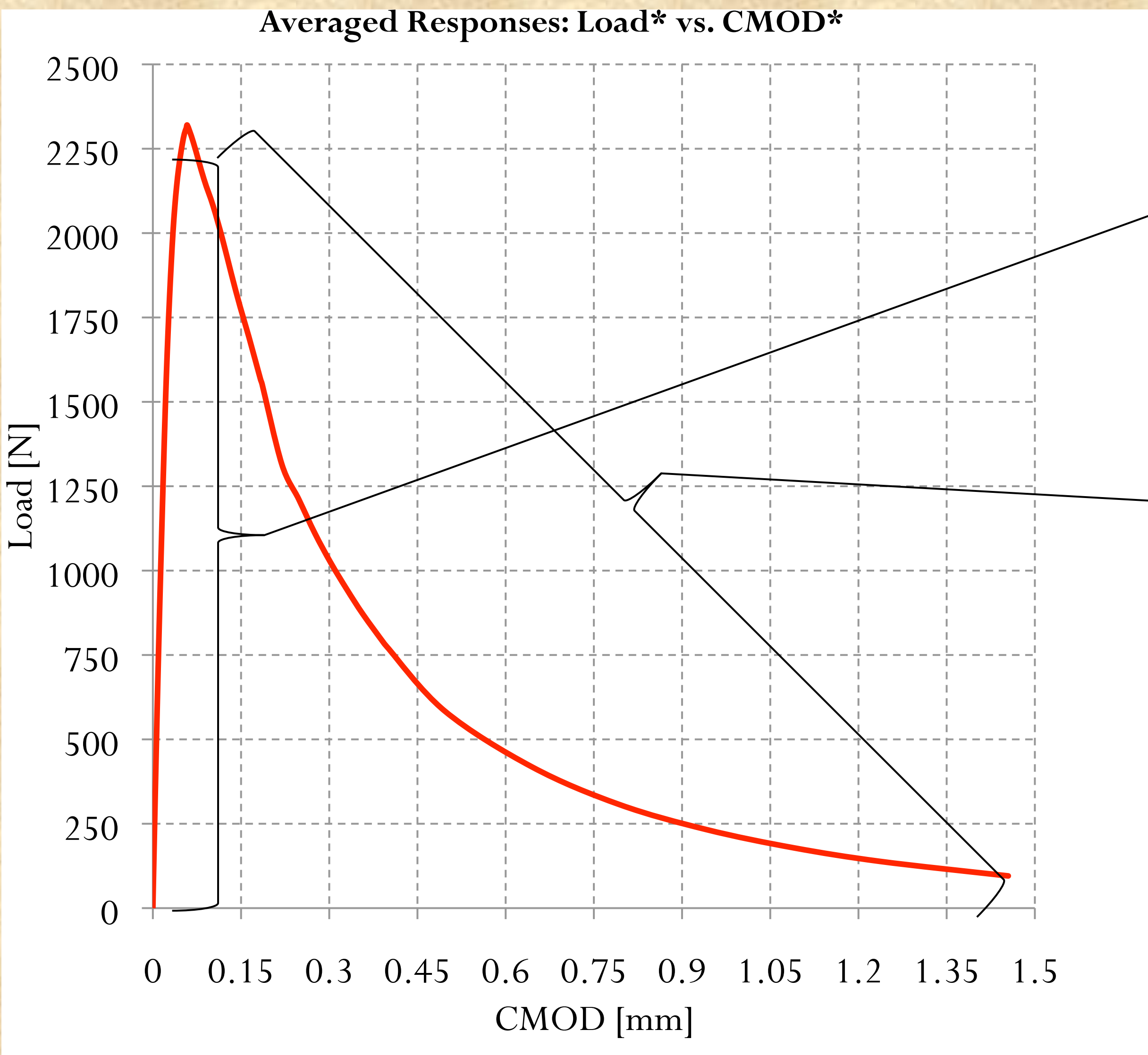
Mechanical testing of *pozzolanic mortar* - traction







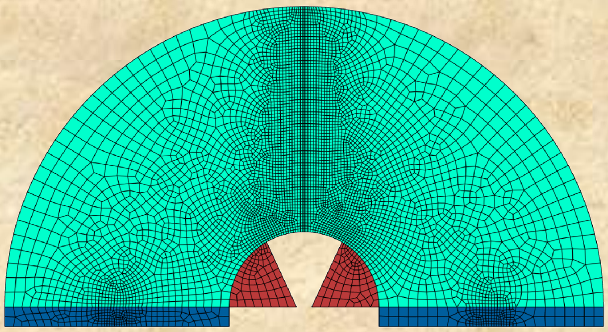
Mechanical testing of *pozzolanic mortar* - traction

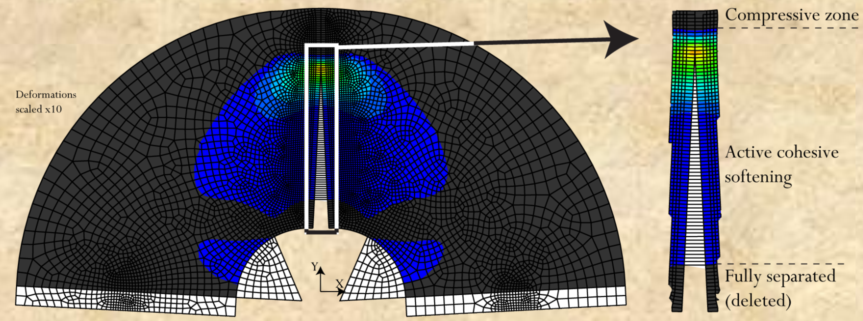


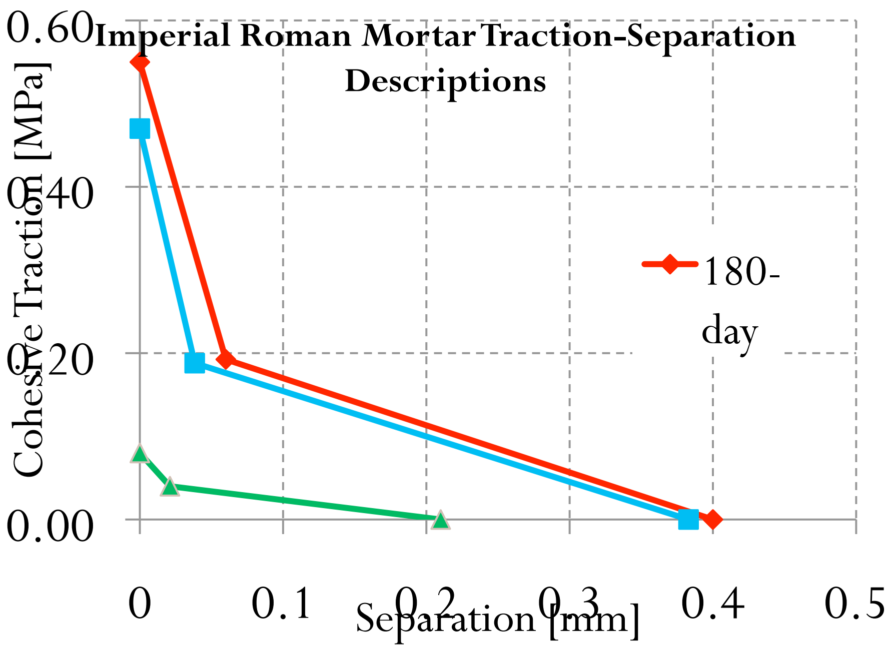
Engineering Properties:
- Young's Modulus
- Modulus of Rupture

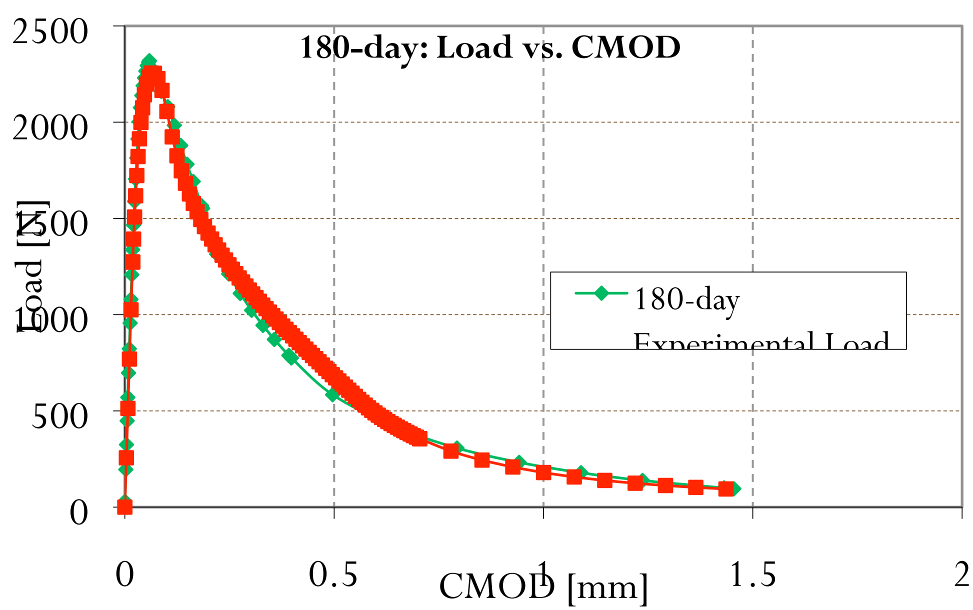
Fracture Properties:
- Tensile strength
- Fracture Energy
- Softening curve form

Fracture properties extraction via FEM simulation

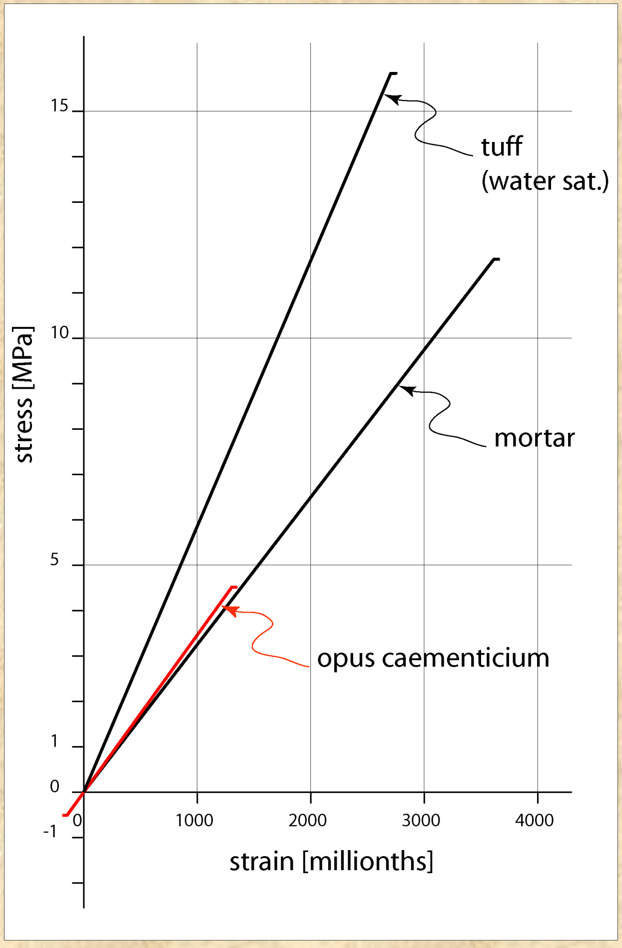








Opus caementicium - linear FEM model



Model assumes identical linear elastic behavior in compression and tension

Elastic modulus = 3 - 3.5 GPa
Ultimate strength in compression = 5 MPa
Ultimate strength in tension = 0.5 MPa
Mass density = 1500 kg/m^3

Appropriate for STATIC ANALYSIS:

-- can predict the onset of critical stress state

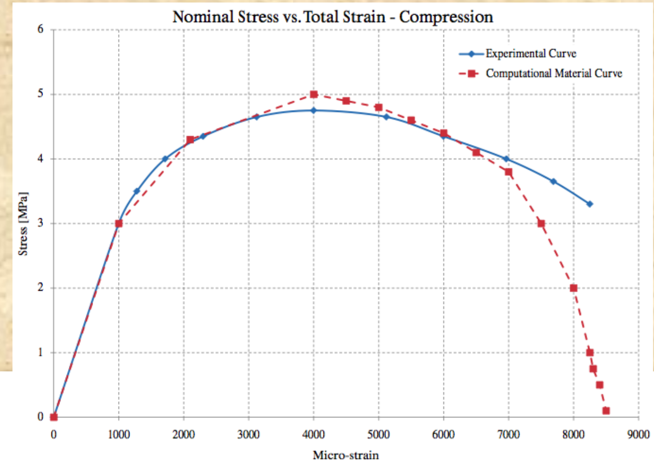
-- but cannot follow the evolution of critical state to collapse

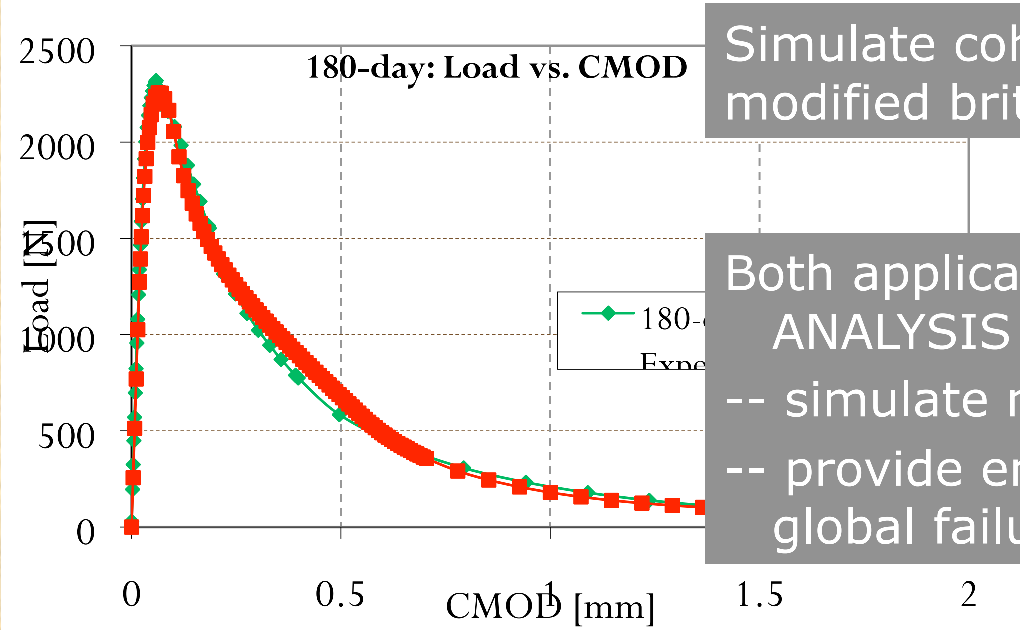
Limited applicability for DYNAMIC ANALYSIS:

-- applicable to modal extraction (natural frequencies)

-- not applicable to advanced earthquake analysis

Opus caementicium - nonlinear FEM models





Concrete damaged plasticity model

Simulates crushing and cohesive fracturing using modified nonlinear plasticity and damage laws

Fracture models

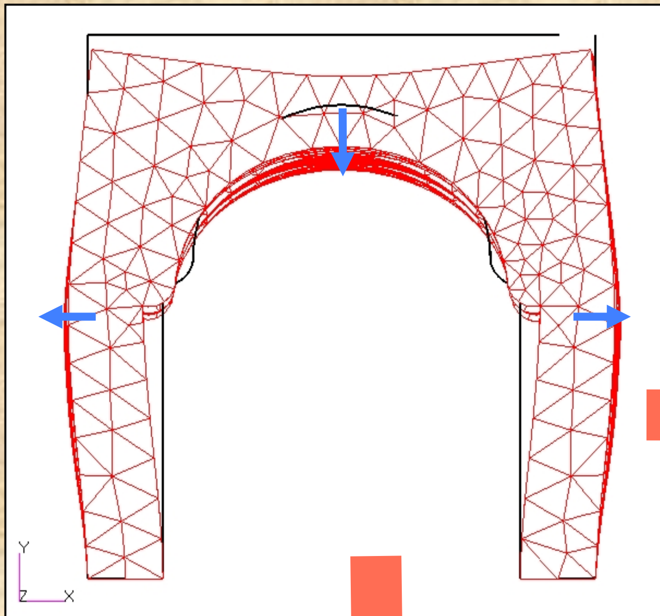
Simulate cohesive fracturing using cohesive or modified brittle fracturing laws

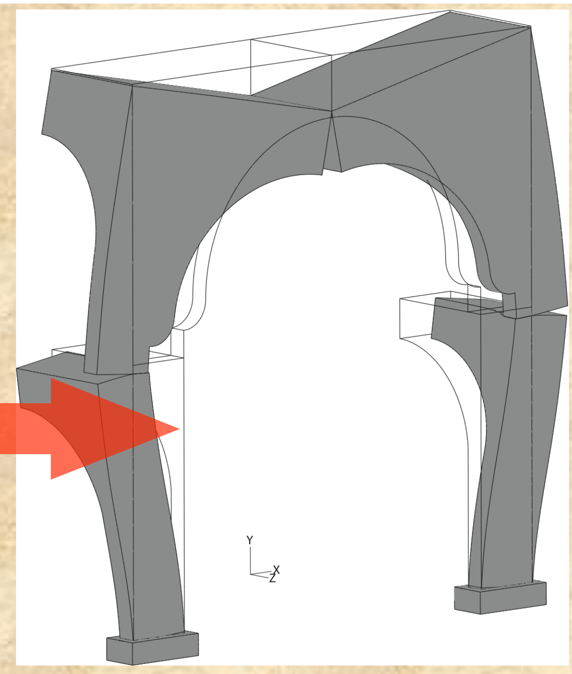
Both applicable for STATIC and DYNAMIC ANALYSIS:

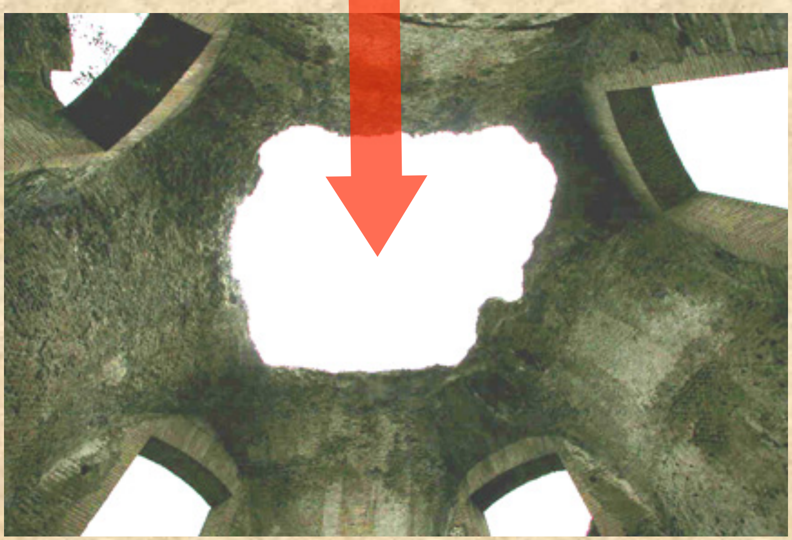
-- simulate material (local) failure

-- provide energy measures for detecting global failure

Opus Caementicium - vault collapse mechanism



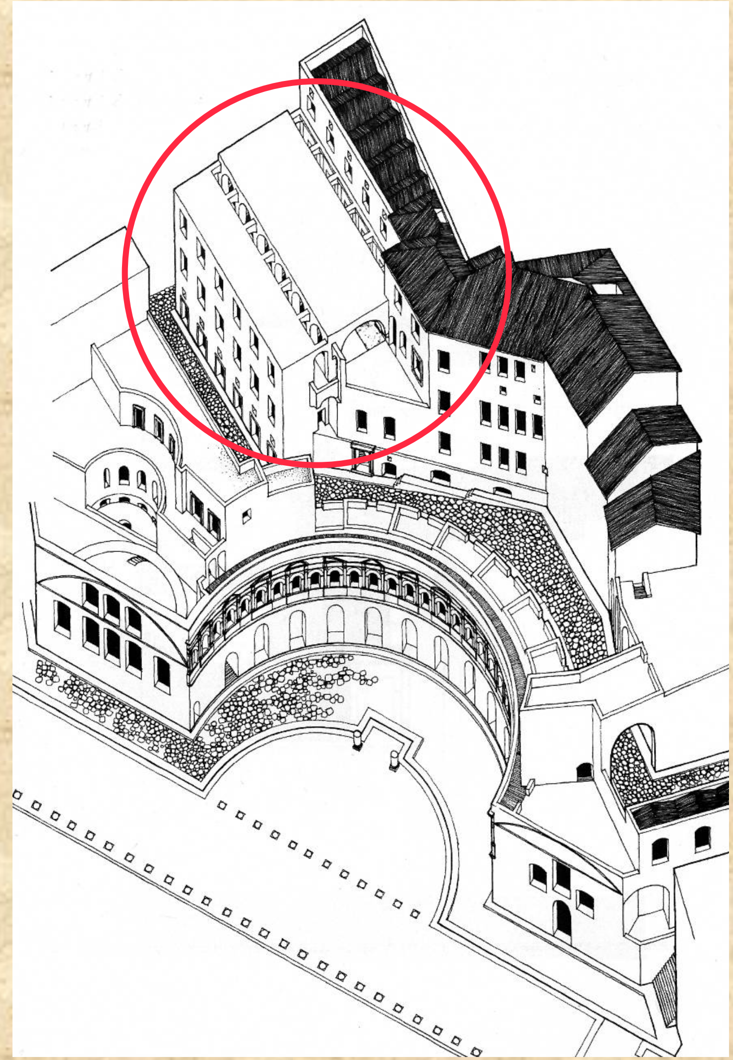




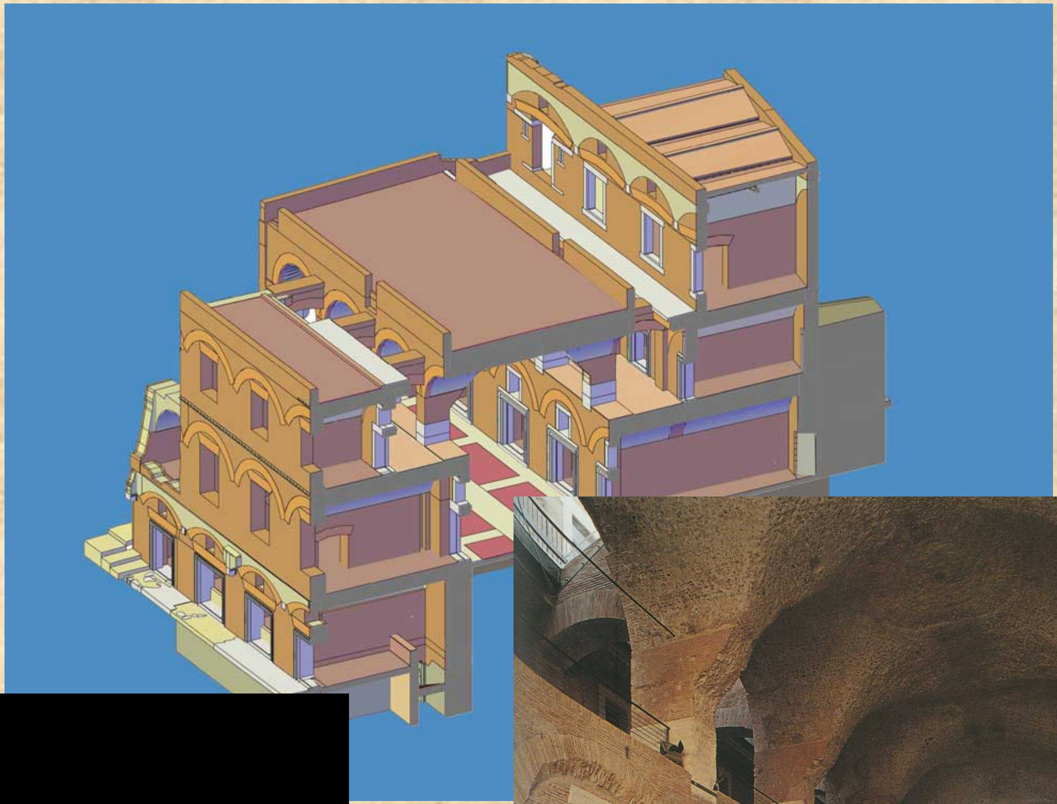
- Fractures weaken the structure by reducing load paths, changing static and dynamic behavior
- Functionality may be compromised
- Fractures may lead to catastrophic collapse

Hadrian's Villa - Small Baths

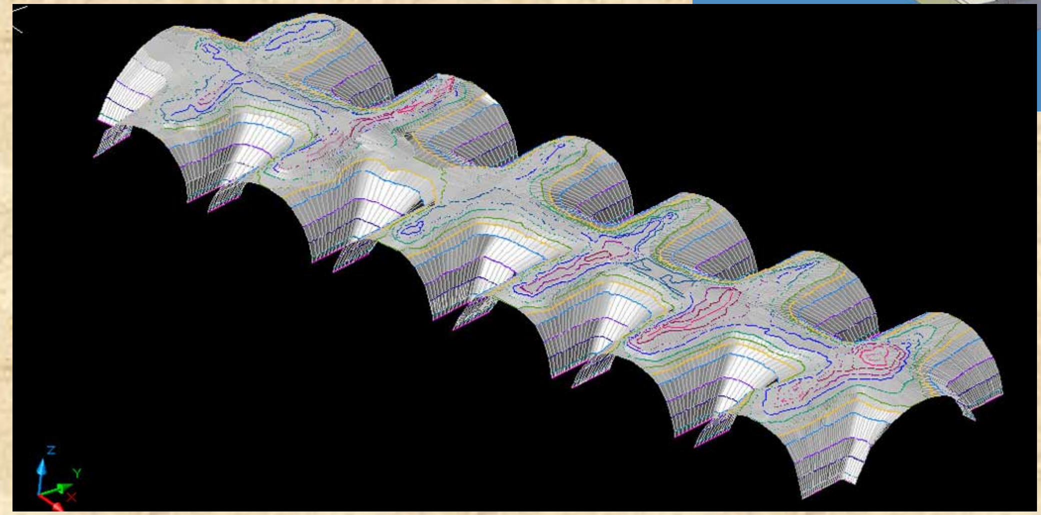
Great Hall of Trajan's Markets (AD 107 - 110)

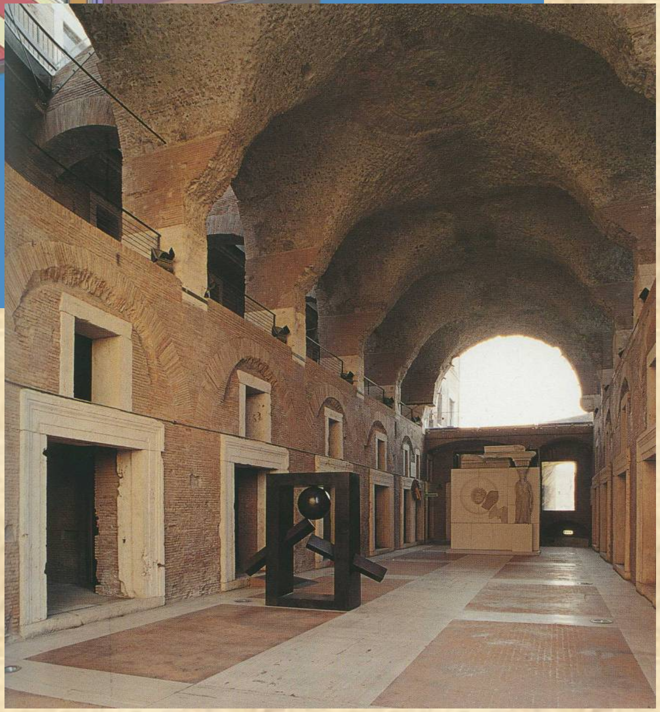


Great Hall - architecture

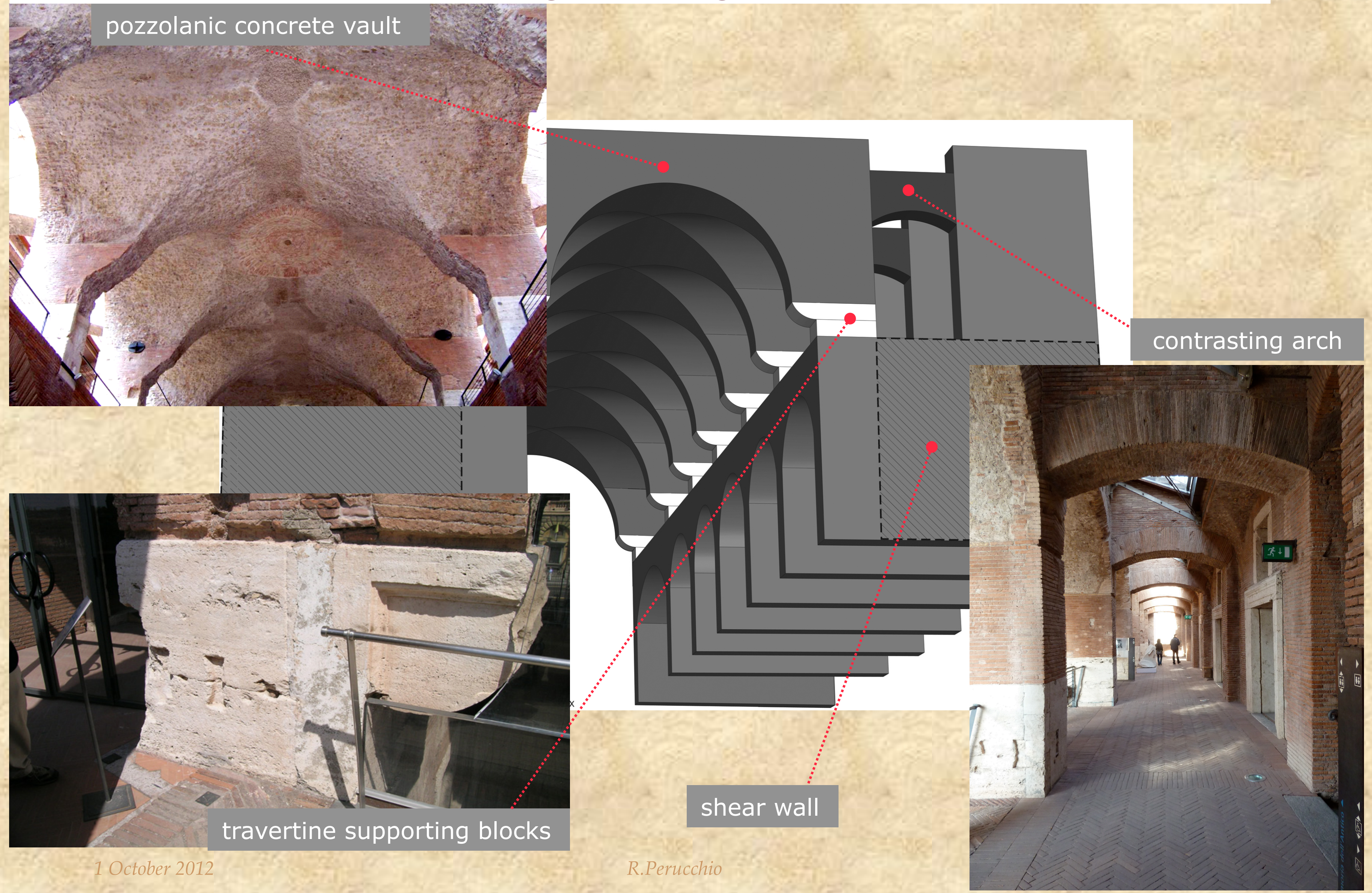


AUTOCAD model by Marco Bianchini
Museo dei Fori Imepriali - Roma





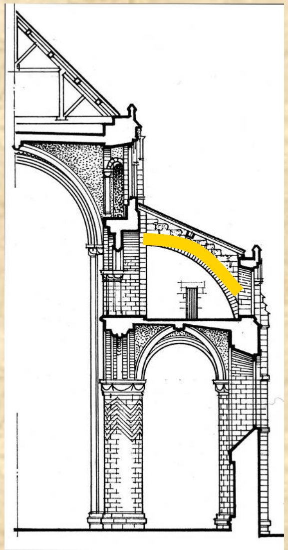
Great Hall - structural engineering



Great Hall structural system

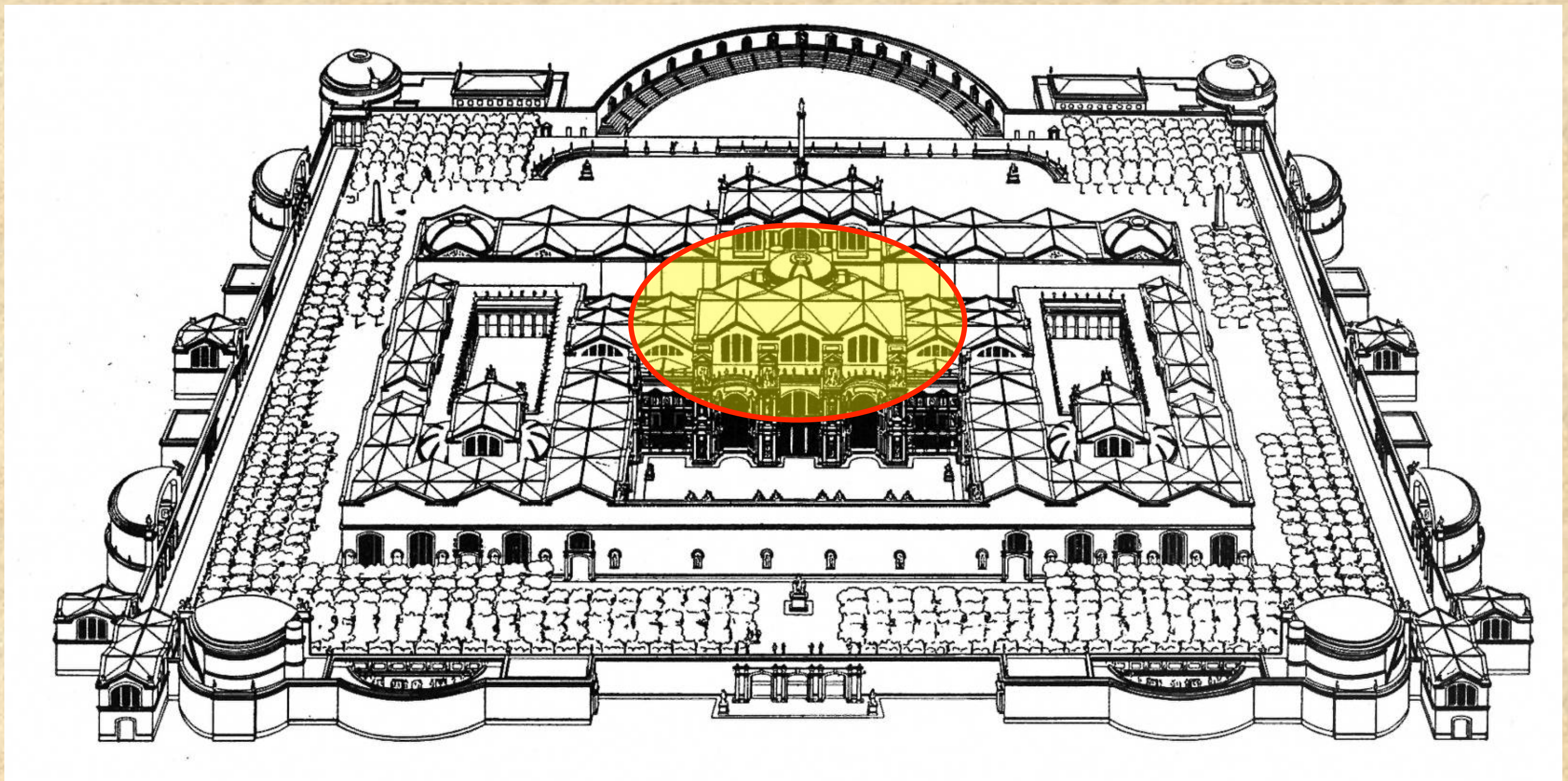
"CONTRASTING ARCHES"

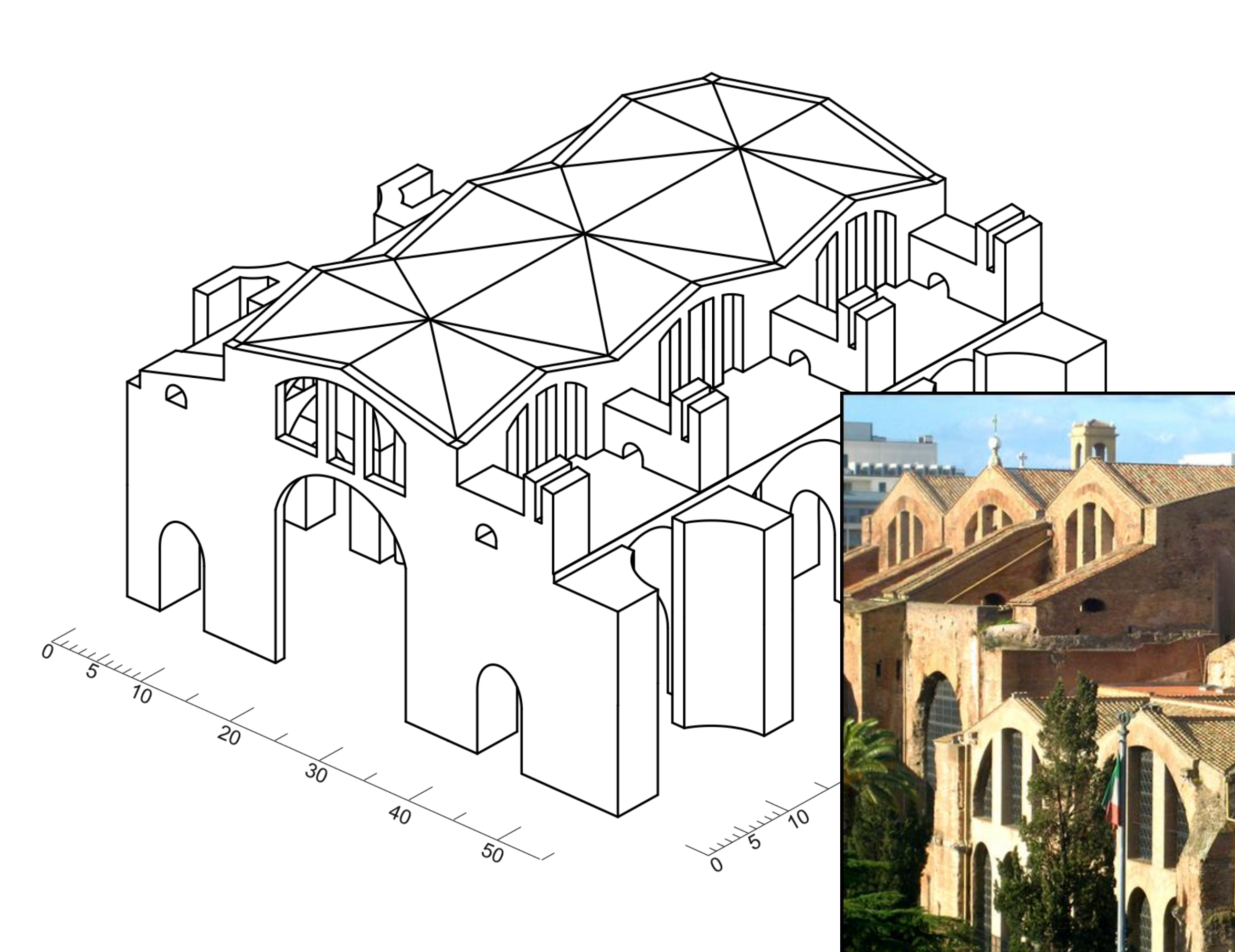
Giovannoni 1913 : prototypes of flying buttresses

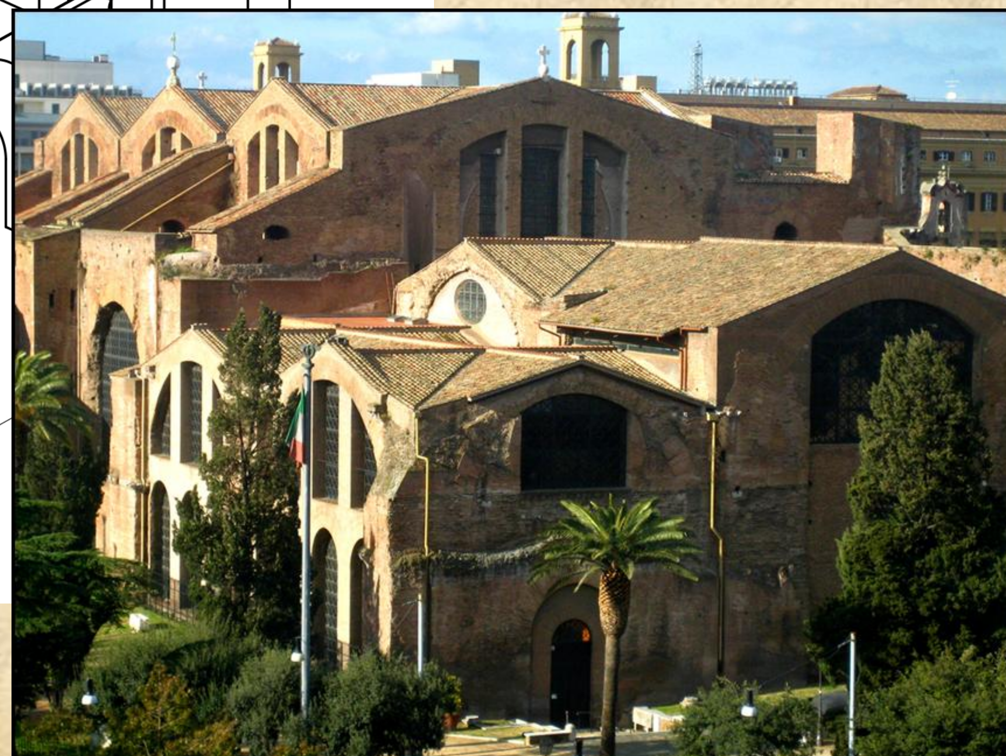


G. Giovannoni, "Prototipi di archi rampanti in costruzioni romane", *Ann. Società Ingegneri Architetti Italiani*, 10, 1913, pp. 279-92.

Baths of Diocletian - (AD 298-305)

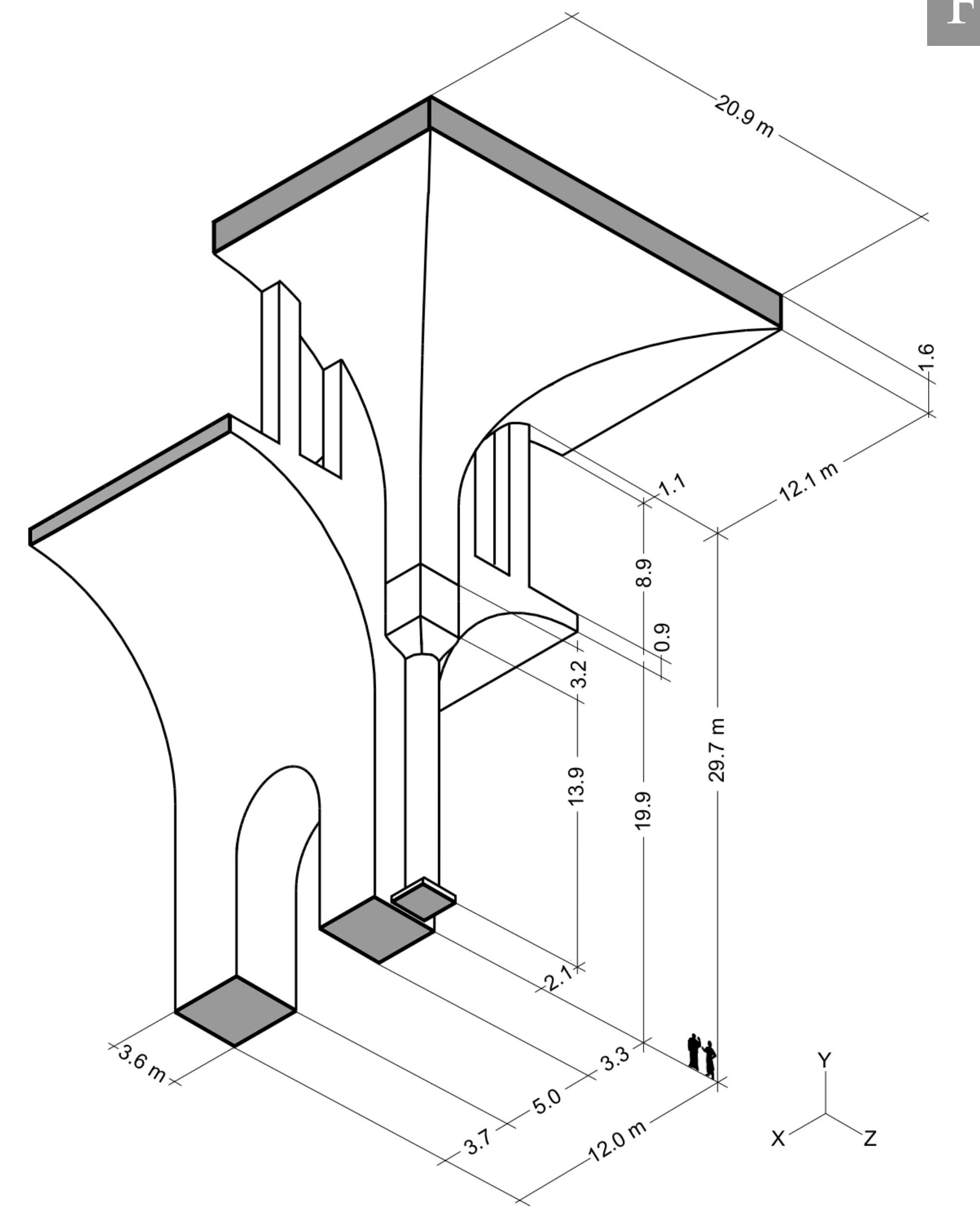






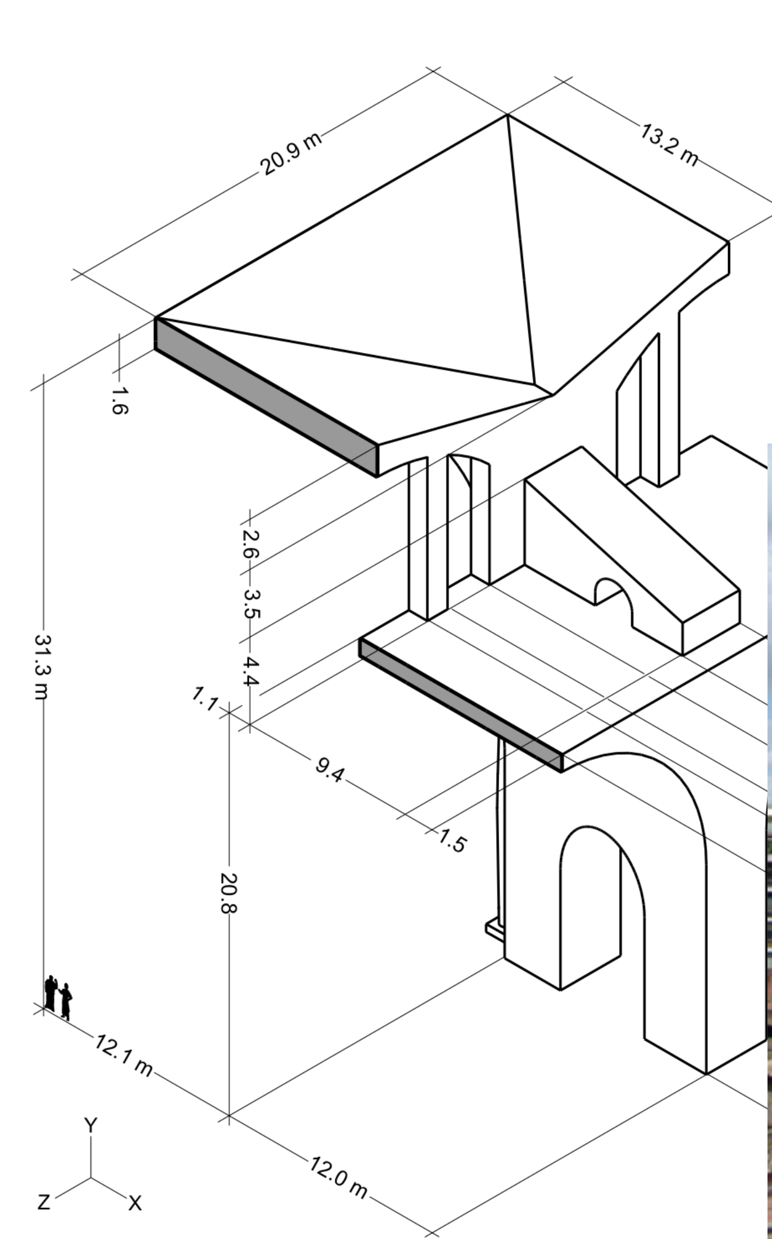
Baths of Diocletian - Frigidarium - exterior

Frigidarium - interior

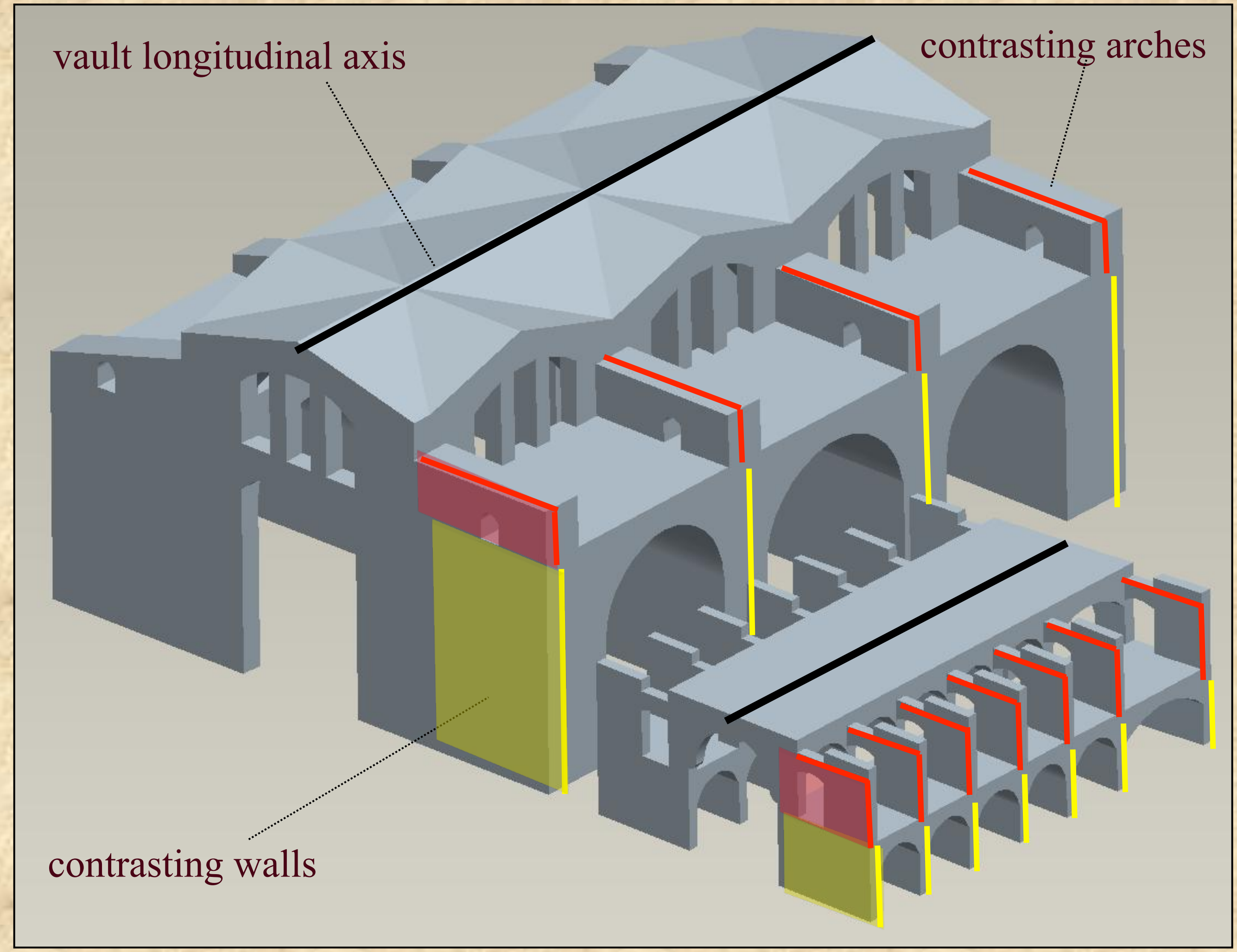


Frigidarium - interior - cross vaults

Frigidarium - exterior contrasting arches

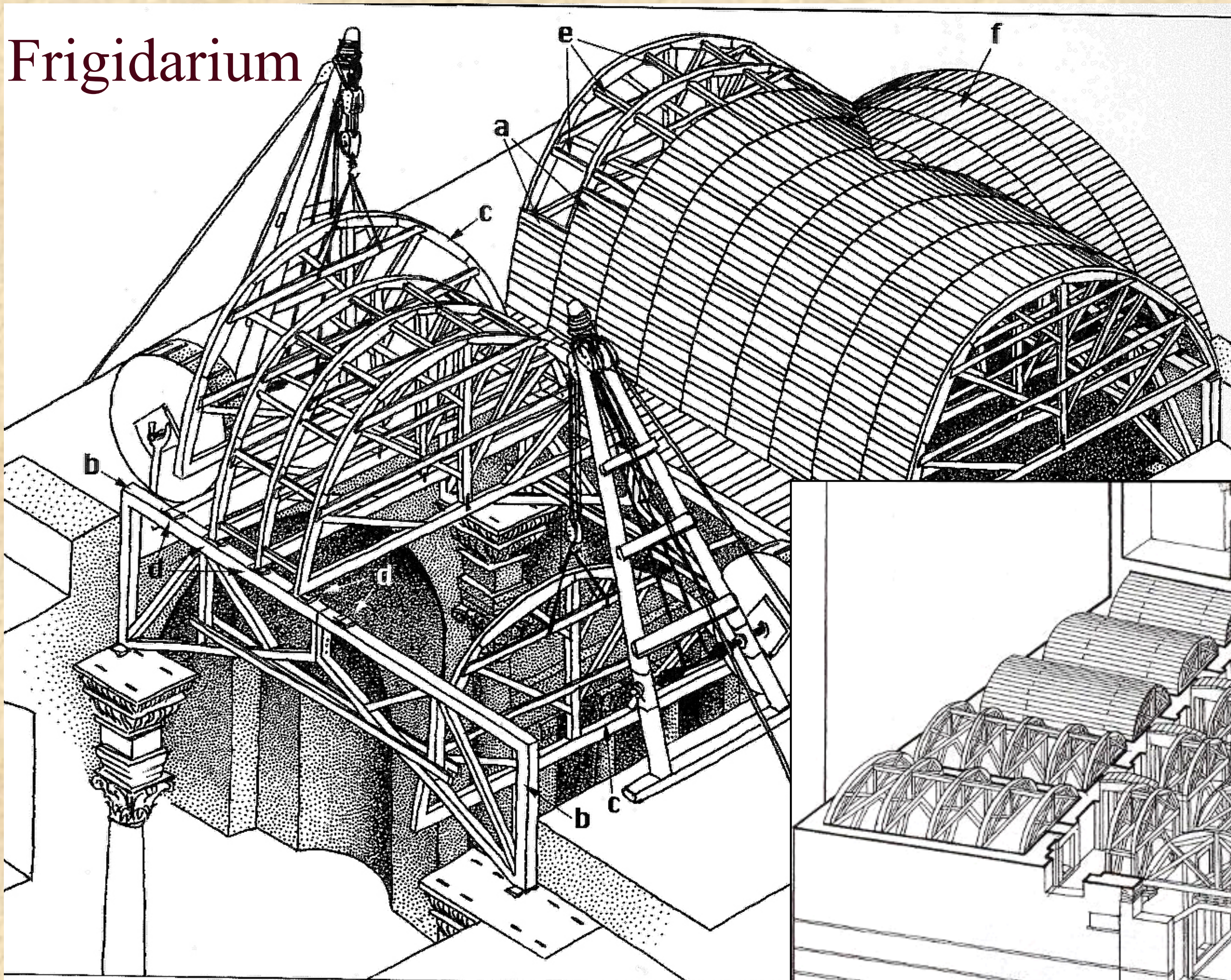


Frigidarium/Great Hall - structural skeletons

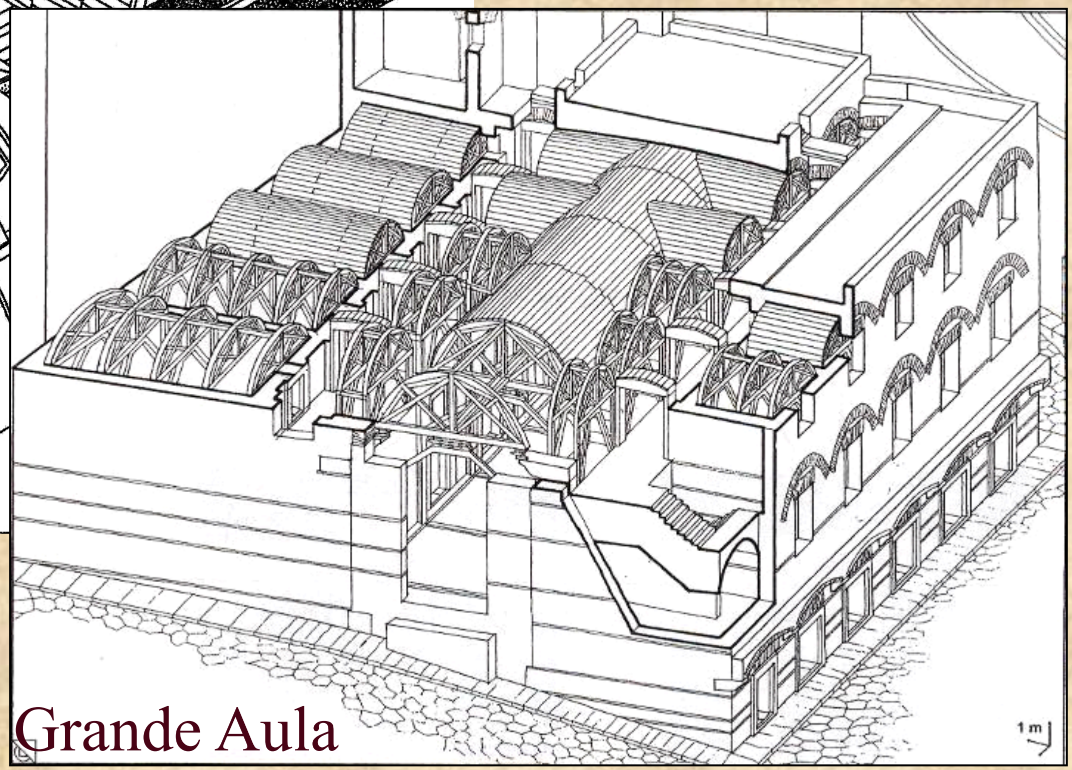


Opus Caementicium - timber frameworks

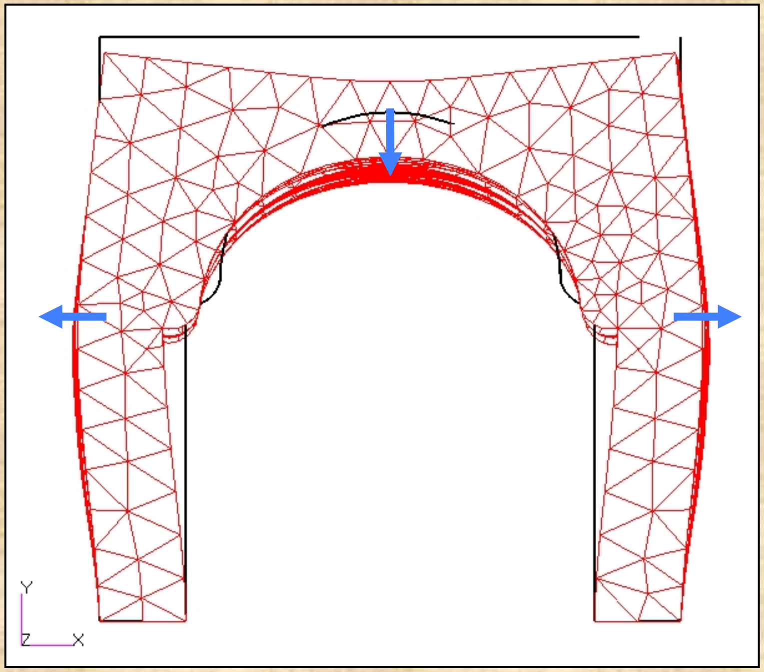
Frigidarium

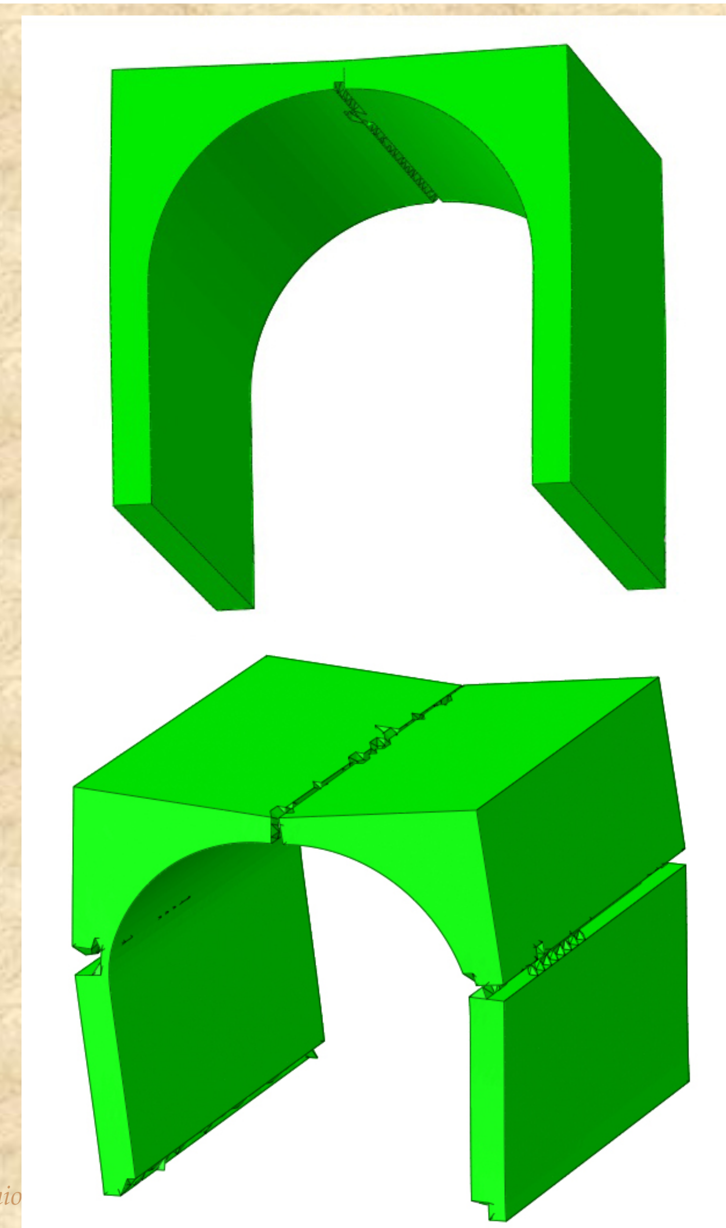


Grande Aula



Opus Caementicium - vault collapse mechanism

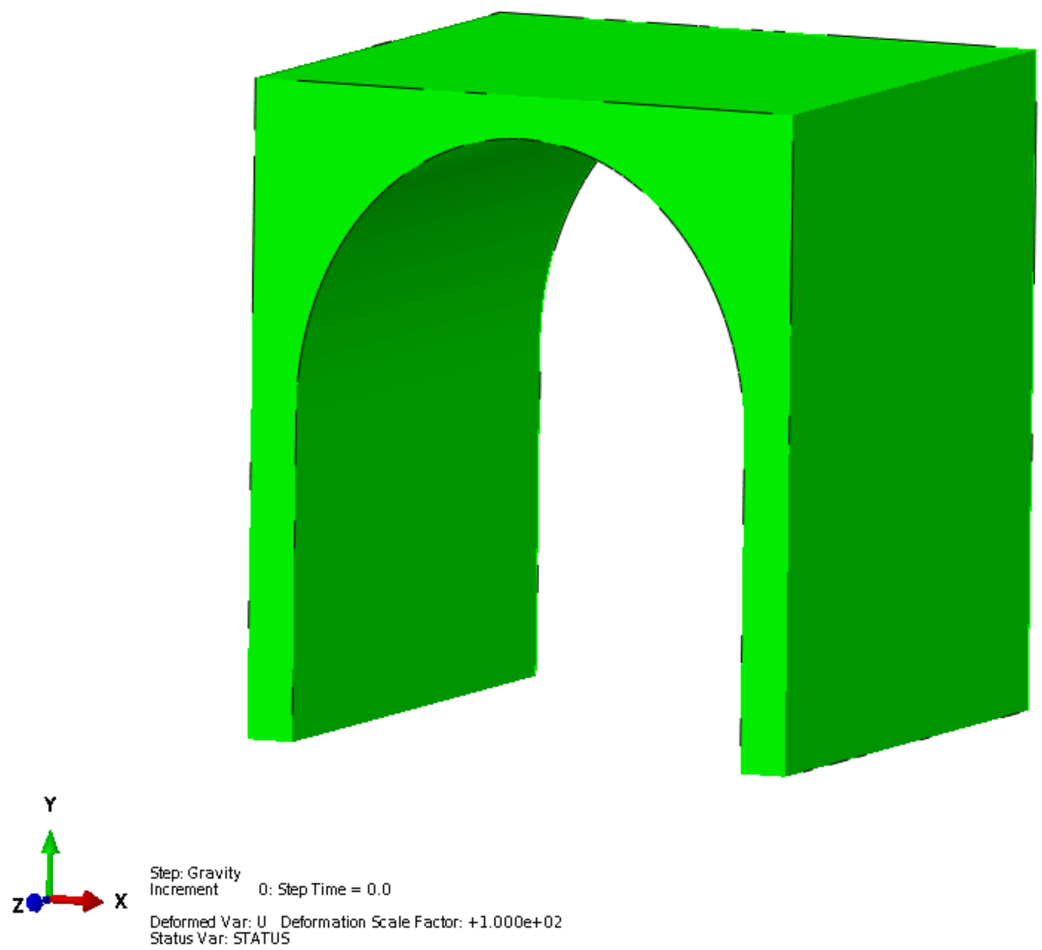




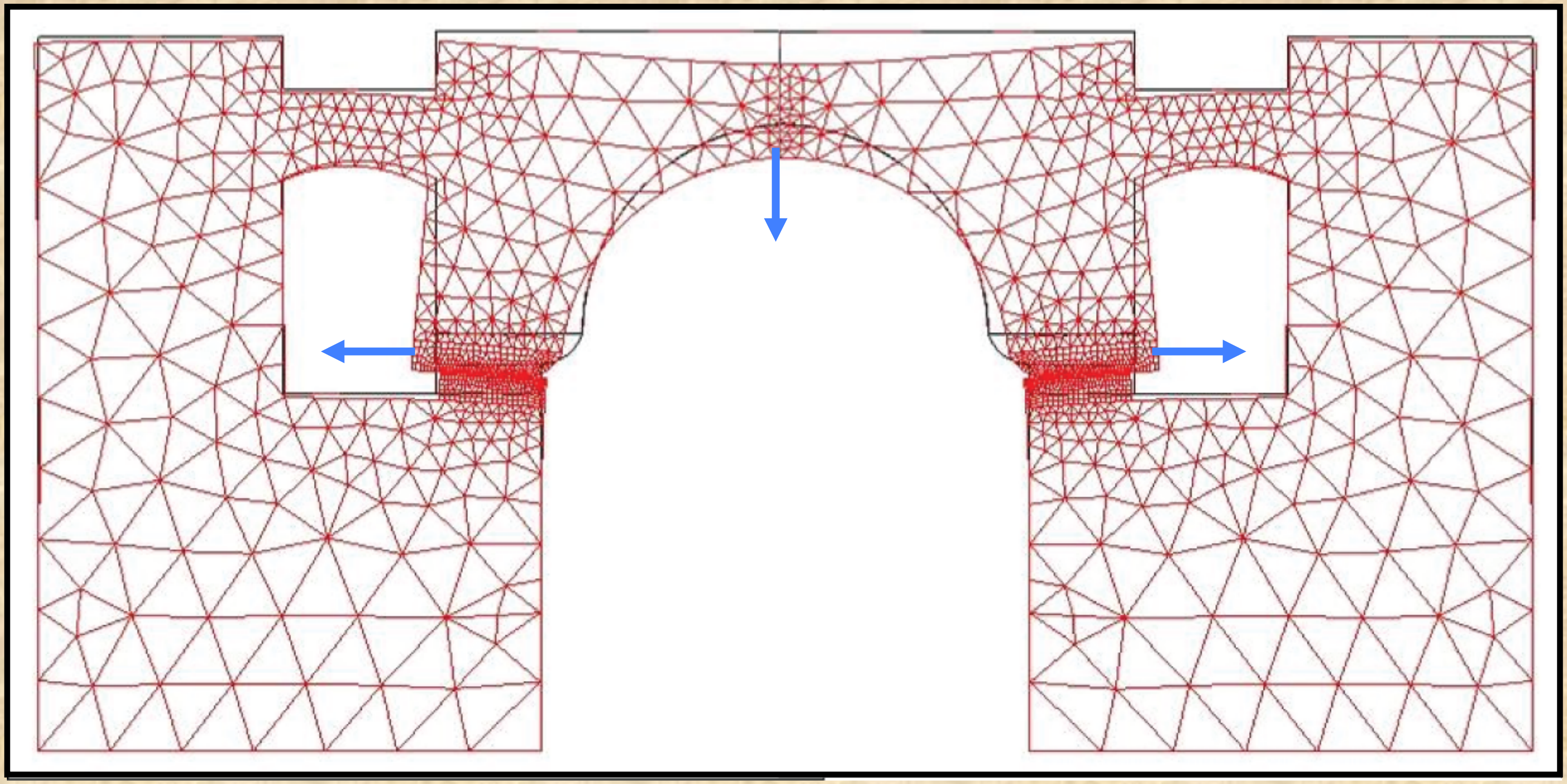
deformation and collapse under 1.5 g gravitational load

Opus Caementicium - vault collapse mechanism

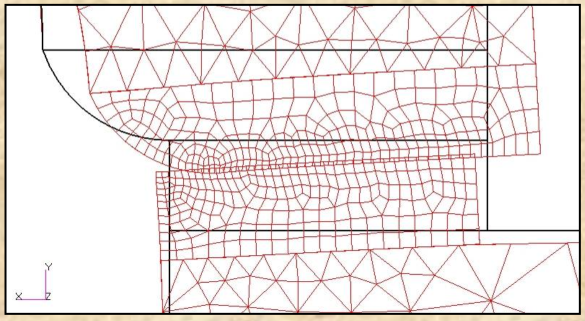
deformation and collapse under 1.5 g gravitational load



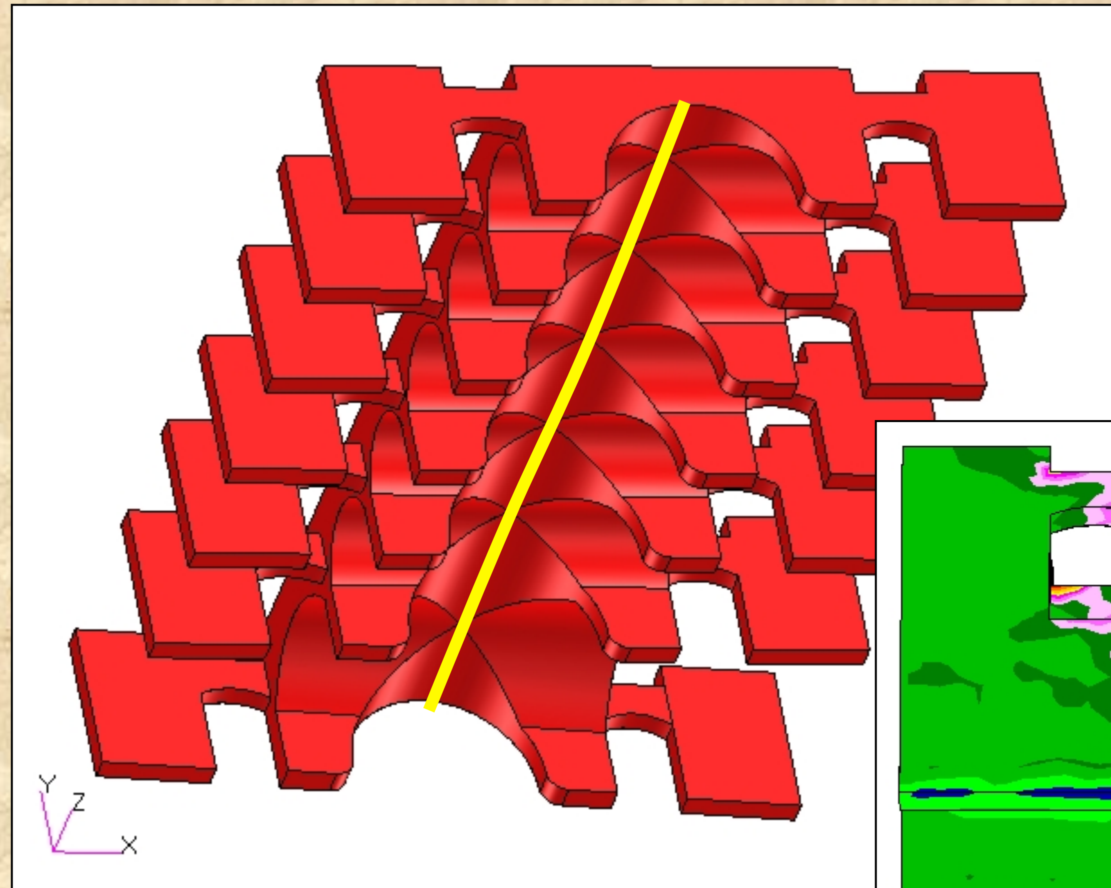
Opus Caementicium - Great Hall collapse mechanism



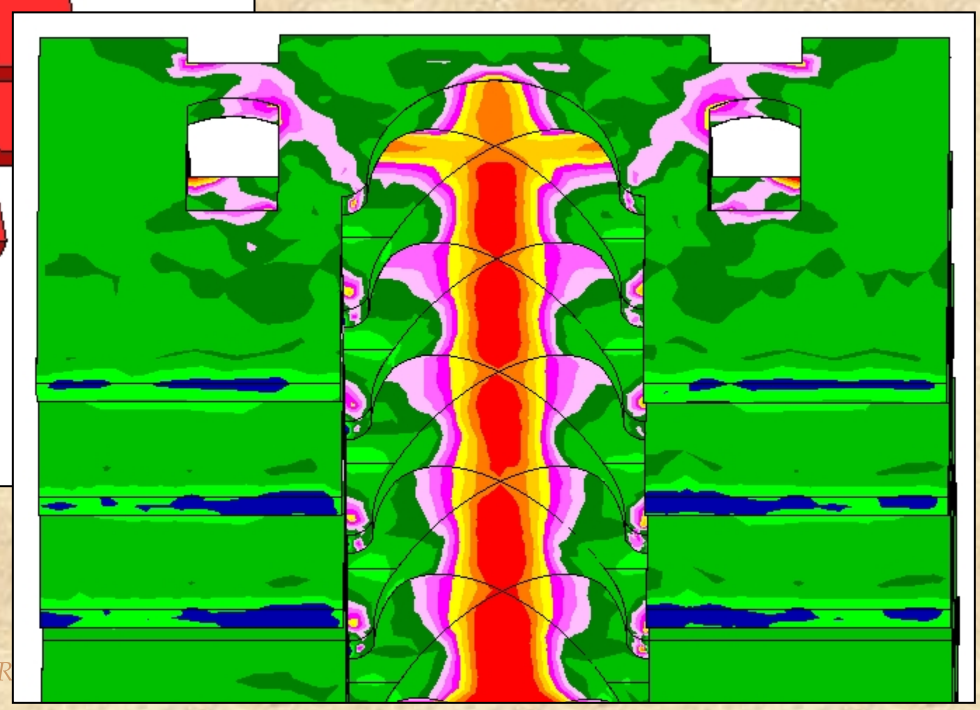
Relative motion of the supporting blocks (travertine) increases downward deformations and tensions at the crown



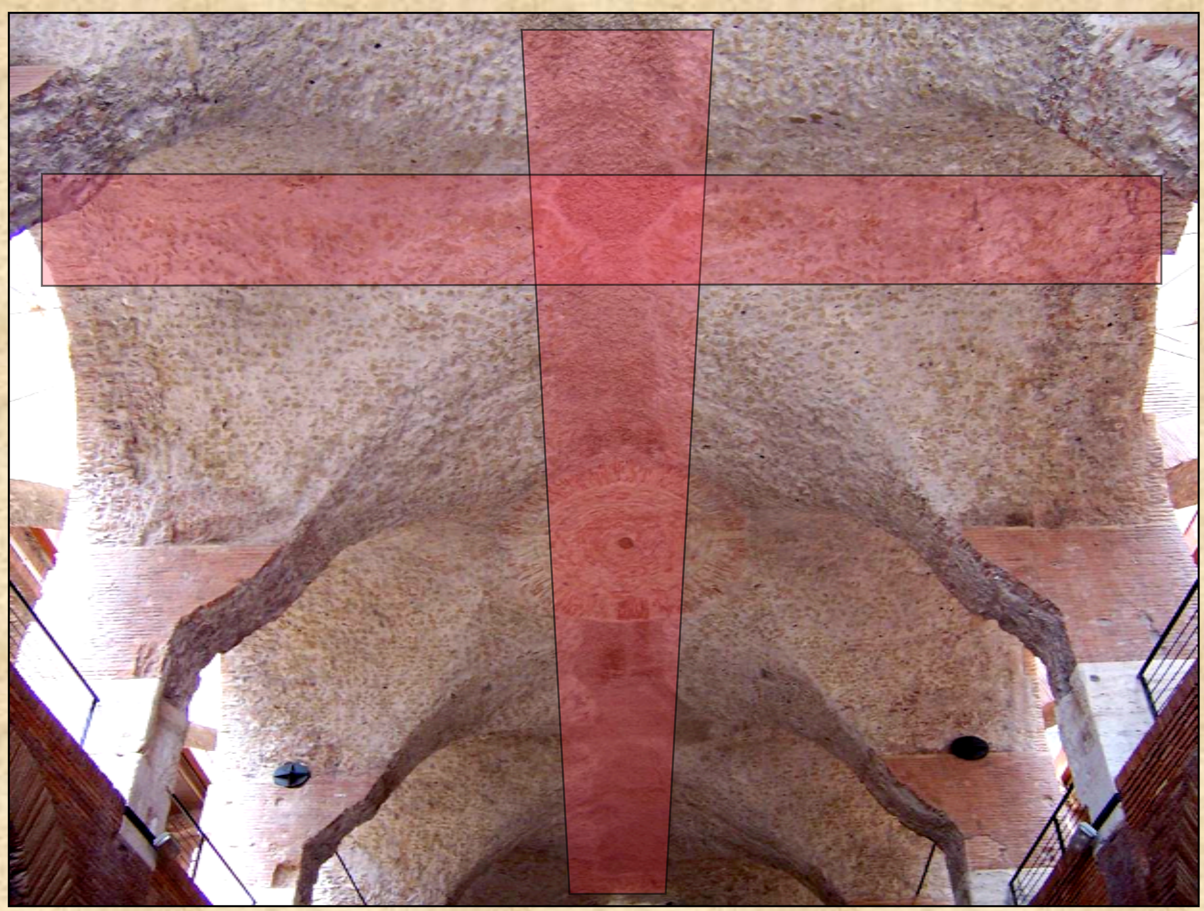
Great Hall - fracture at vault intrados



High tensile stresses may cause a longitudinal crack at the intrados of the crown

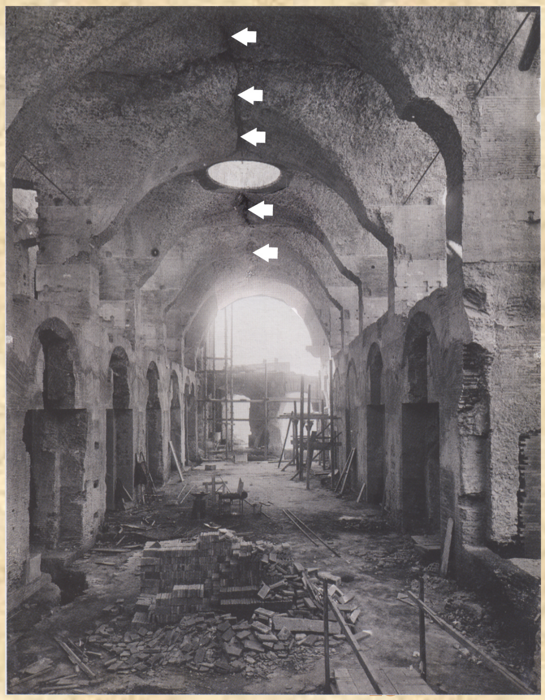


Great Hall - fracture at vault intrados



Excellent correlations between tensile stresses (model) and repaired cracks (reality)

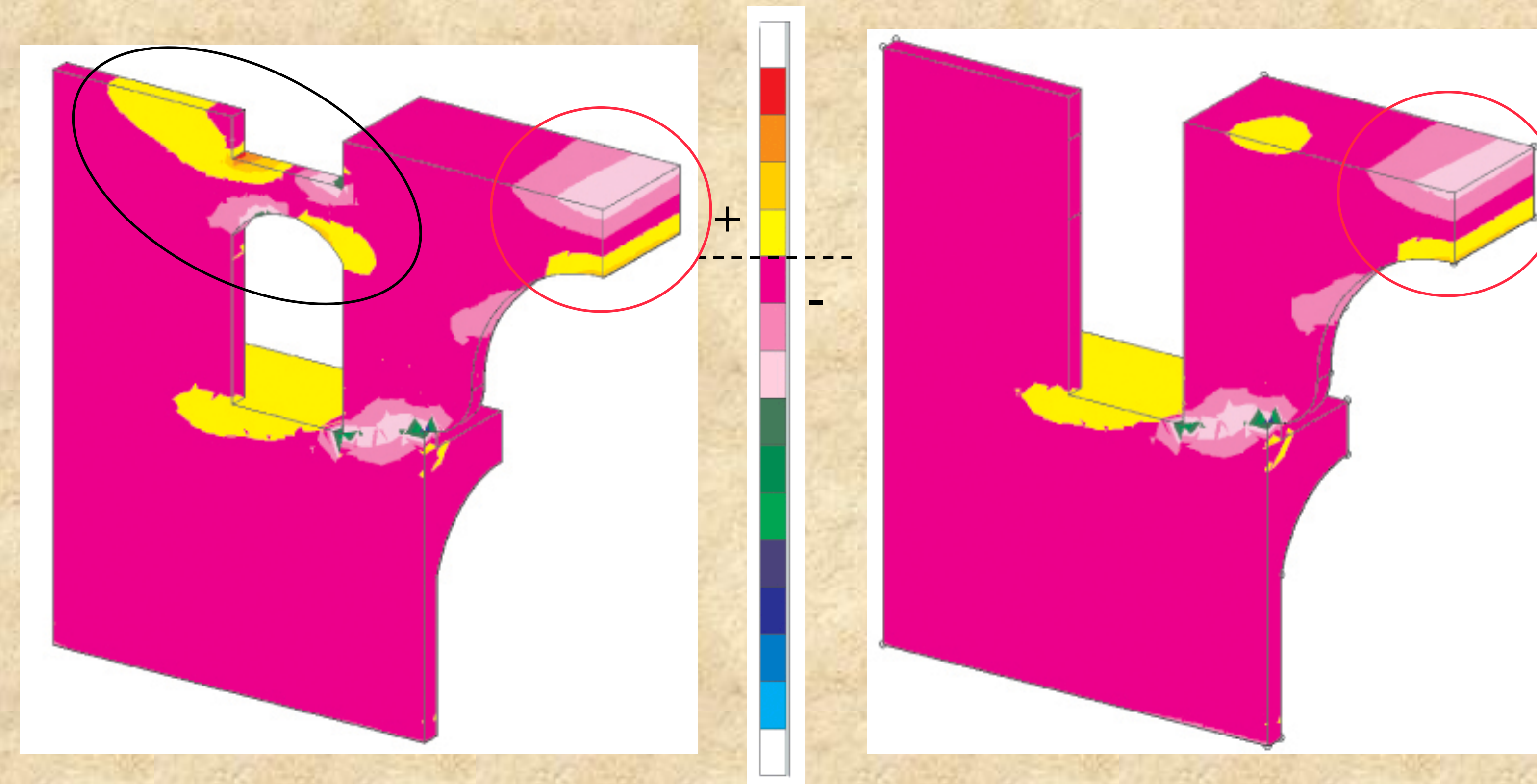
Similar fracture patterns on the lateral vaults



Great Hall - present restored state - 2007

Great Hall - as revealed in 1926 -1934

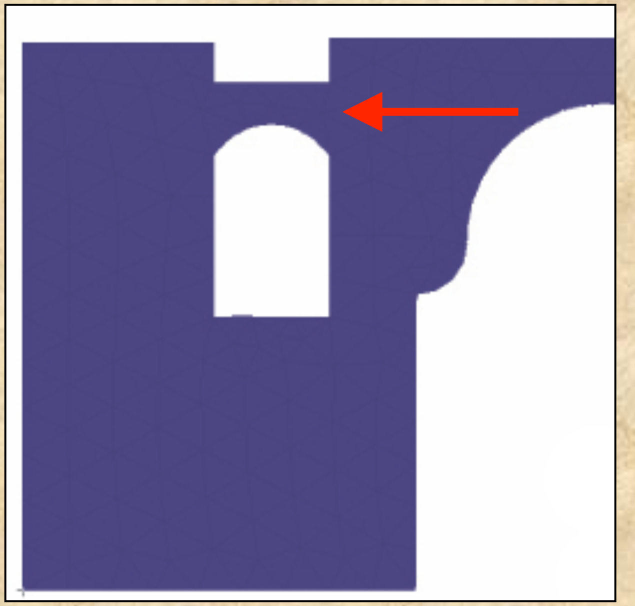
Great Hall - contrasting (?) arches



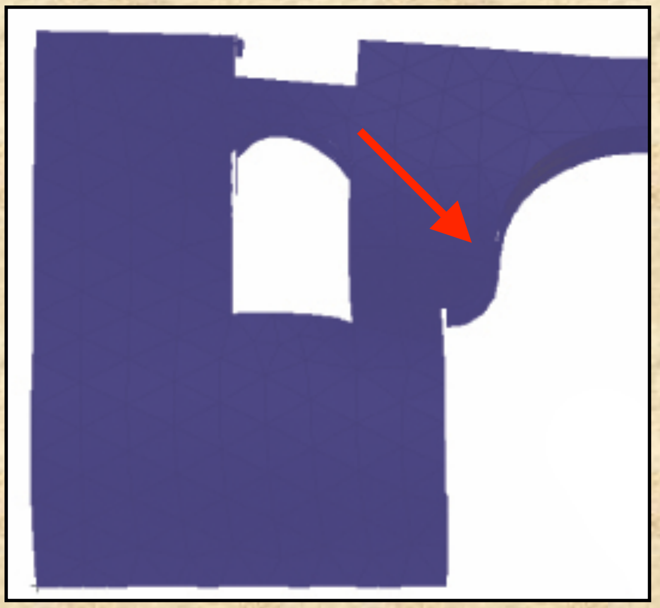
- Contrasting arch does not affect stresses (tensions) at the crown of the vault (under static loading)
- Removing the arch has only a local effect on stress distribution

Great Hall - contrasting (?) arches

- The expected function of the contrasting arch is to prevent the wall - and the vault - from rotating <u>outward</u>



Expected lateral force from vault

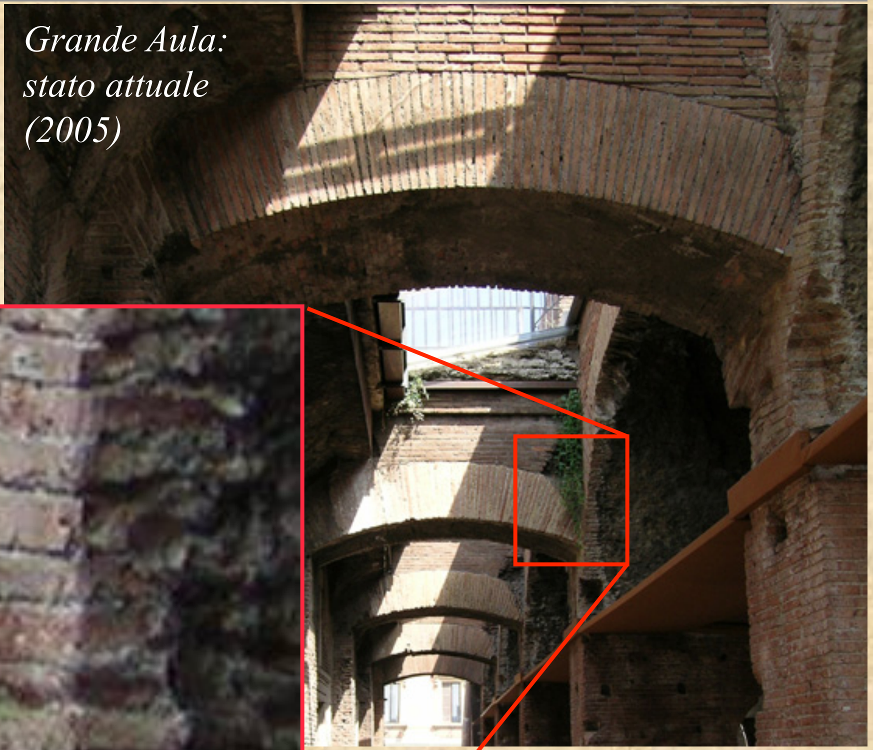


Actual force transmitted by vault

- Under gravitational load the vault rotates <u>inward</u> and pulls down the arch (no contrasting action!)

Great Hall - contrasting (?) arches

Computed stress fields show high tensile stresses at the attachments of the arch



Fractures at the arch springing and evidence of major reconstruction are visible on the actual arches

Great Hall - dislocation of supporting blocks

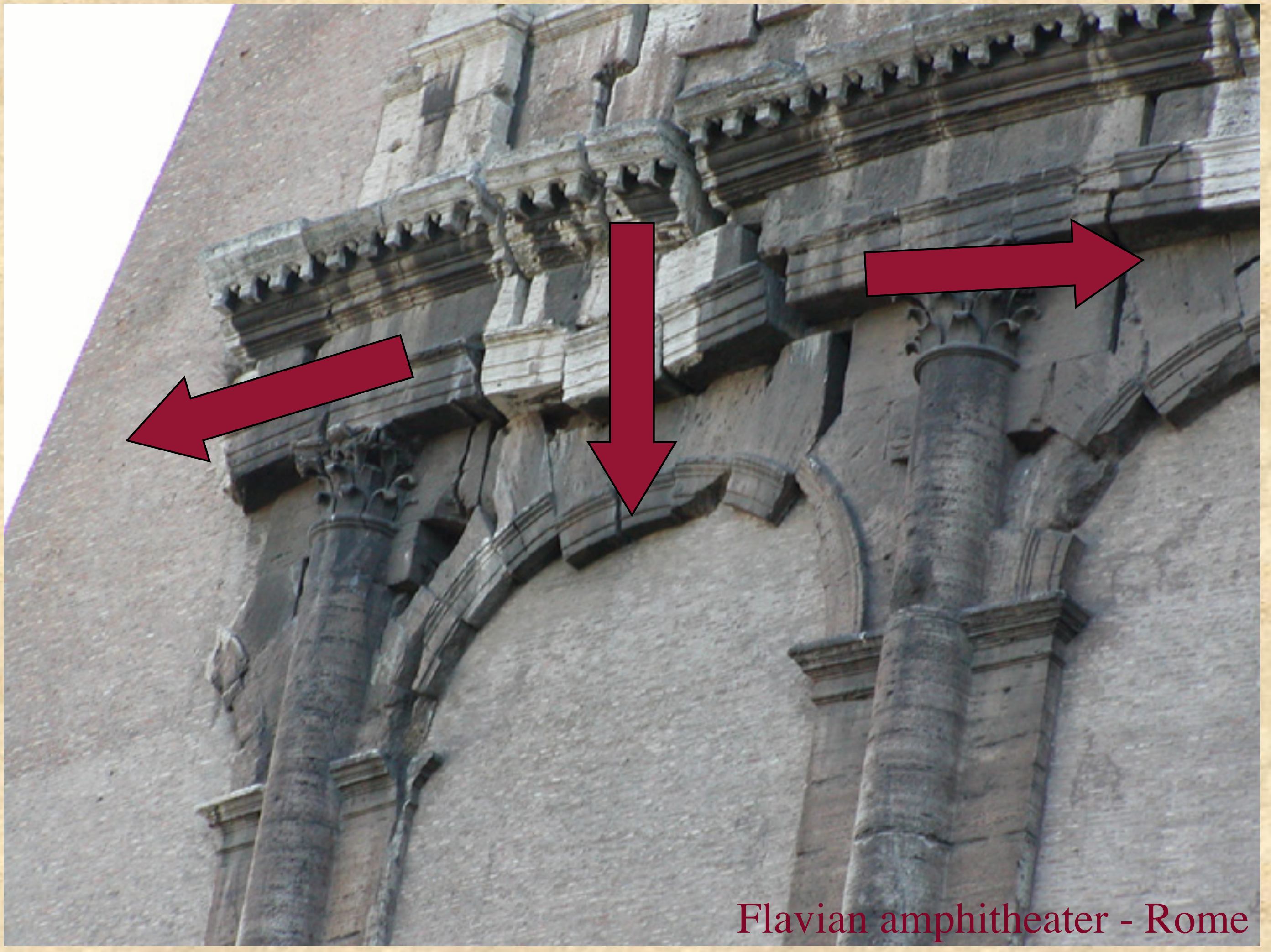
- Signs of dovetail clamps (Roman?) on all blocks
- Blocks are damaged near clamps' placement

- Evidence suggests that clamps were intended to prevent the inward rotation of the blocks

Great Hall - evaluation of structural design

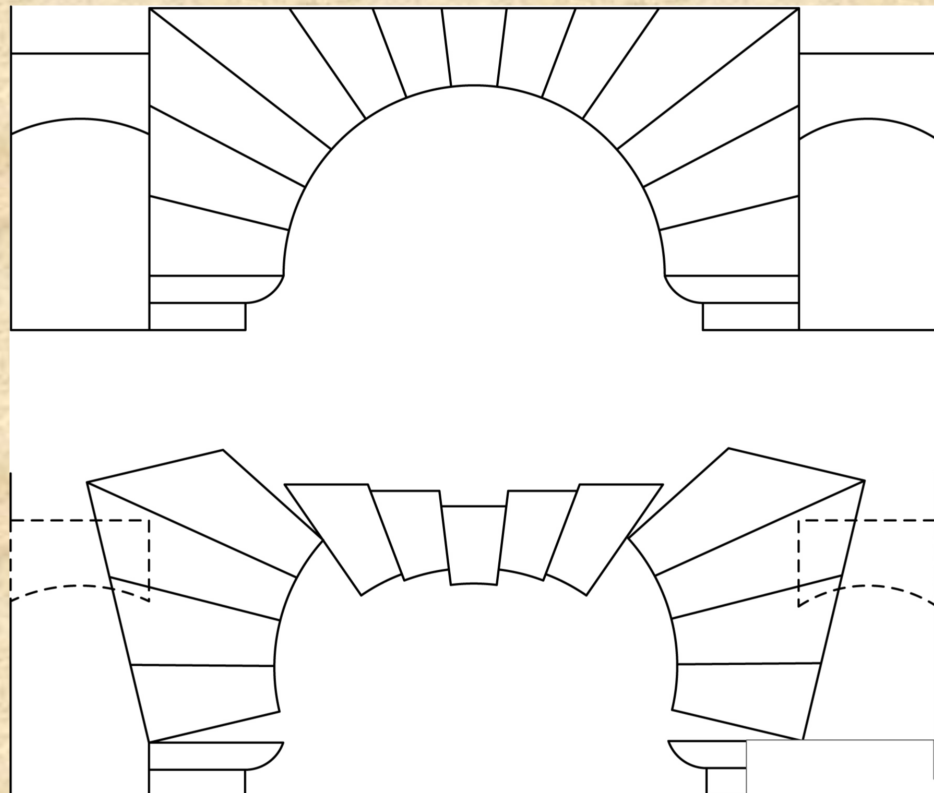
- The Great Hall is an early cross-vault design in *opus caementicium* derived from previous building practices (acquired by building with different materials?)
- This design of the Great Hall is not suitable (produces structural failures) for larger scale cross-vaults (such as the Frigidarium)
- The analysis and the correction of the structural deficiencies revealed by the Great Hall provided the basis for a new (and mature) cross-vaulting design (Frigidarium)

Stone Arch - collapse



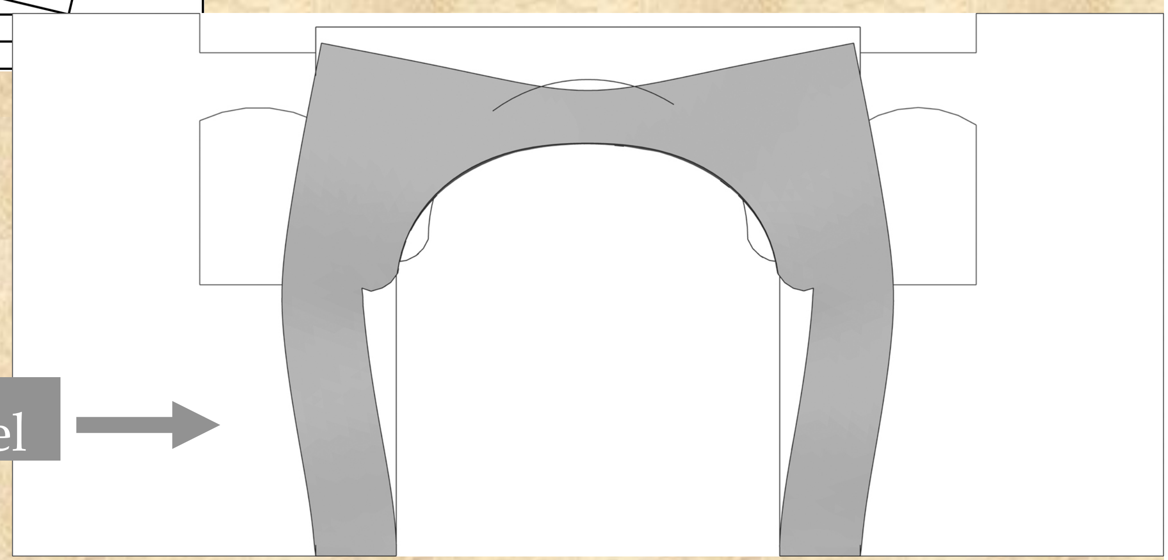
Flavian amphitheater - Rome

Great Hall - vault collapse mechanisms

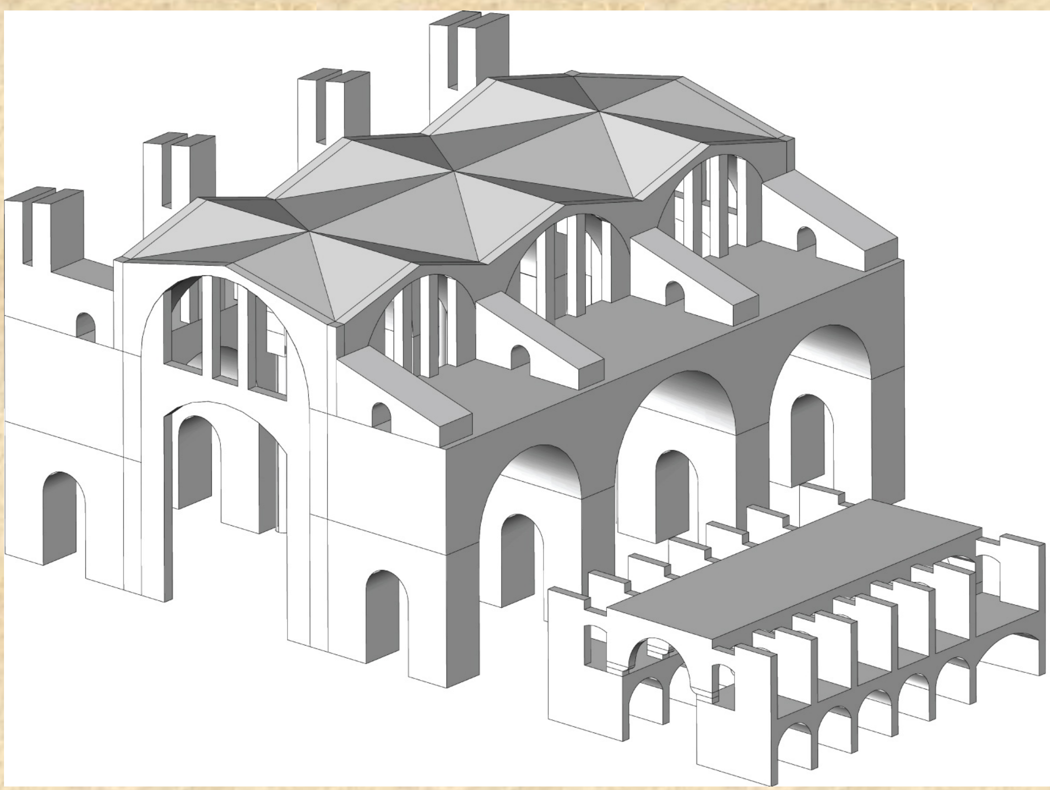


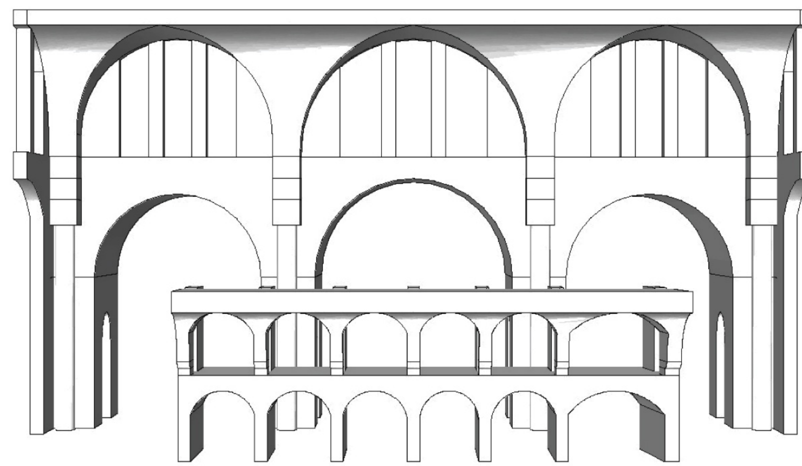
As predicted by dressed stone model

As predicted by concrete model

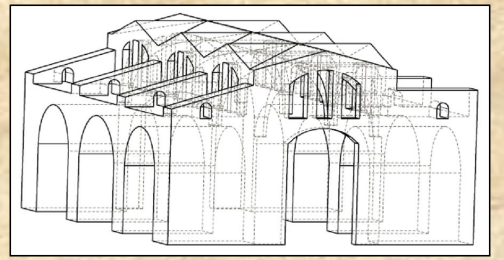


Frigidarium/Great Hall - dimensions





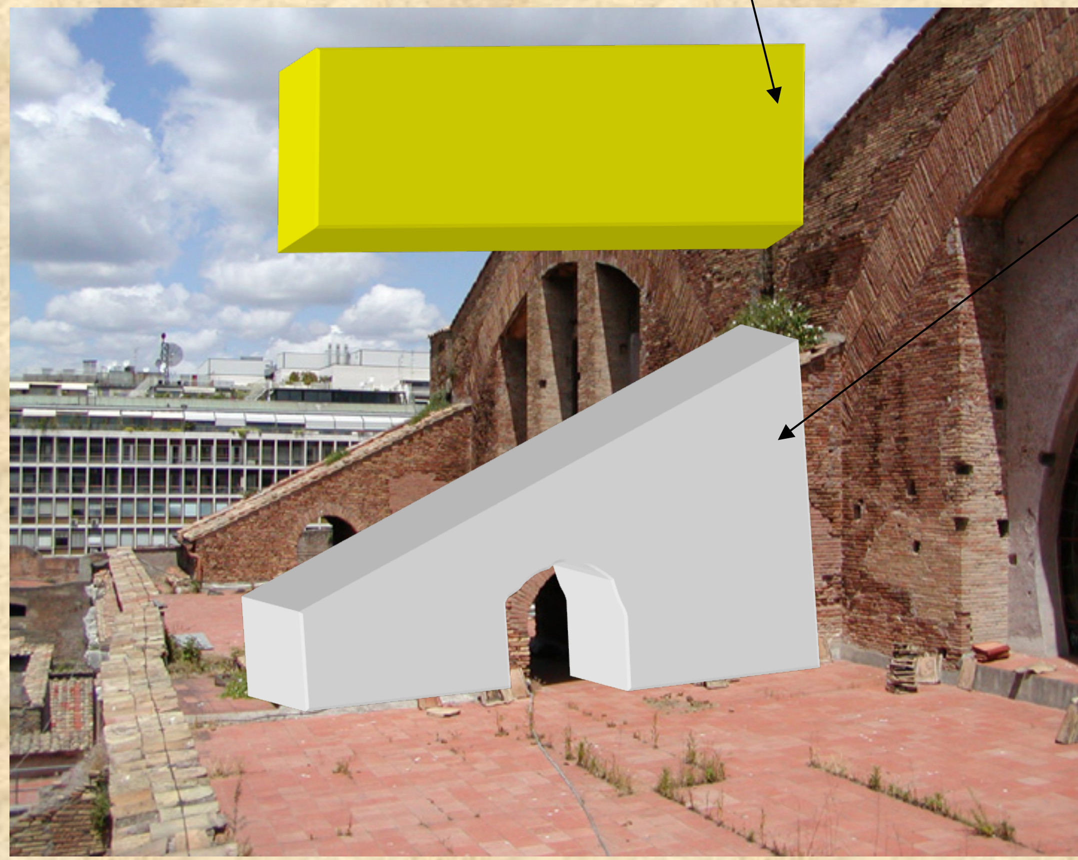
Frigidarium - contrasting arches

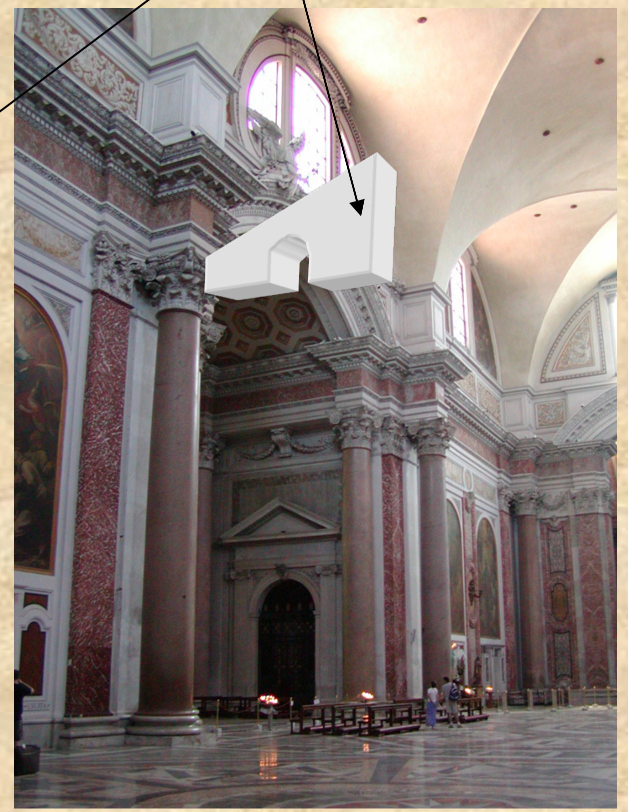


Contrasting arch positioned closer to impost of cross vault, on top of shear wall

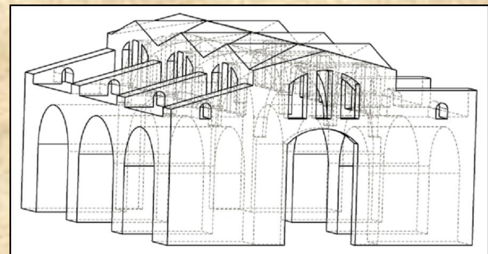
position of arch in Great Hall

position of arch in Frigidarium





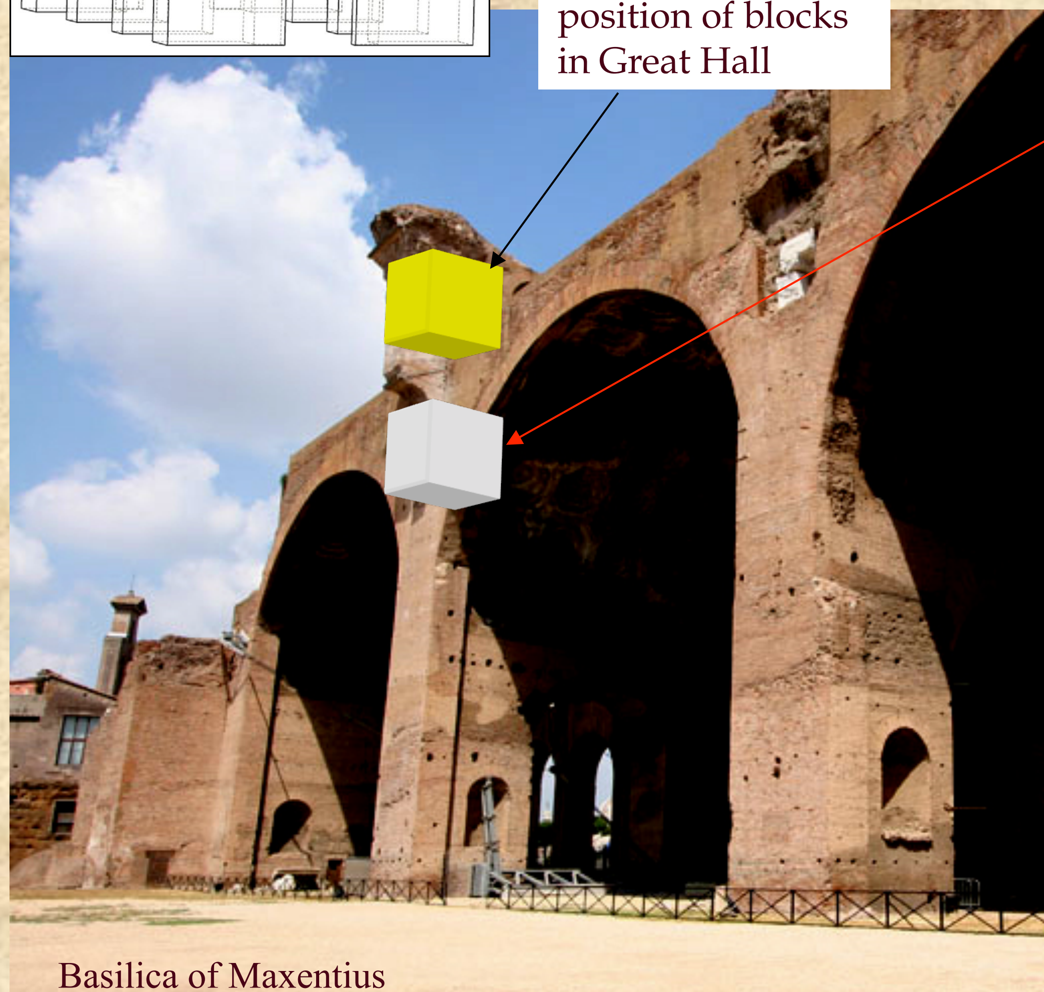
Frigidarium - supporting blocks



Blocks fully embedded (and constrained) in the shear wall

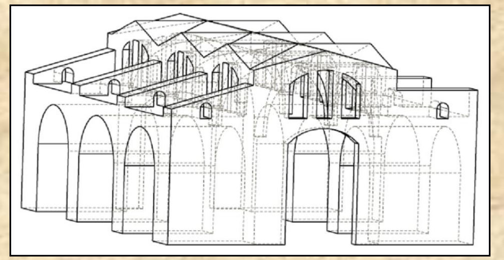
position of blocks in Great Hall

position of blocks in Frigidarium



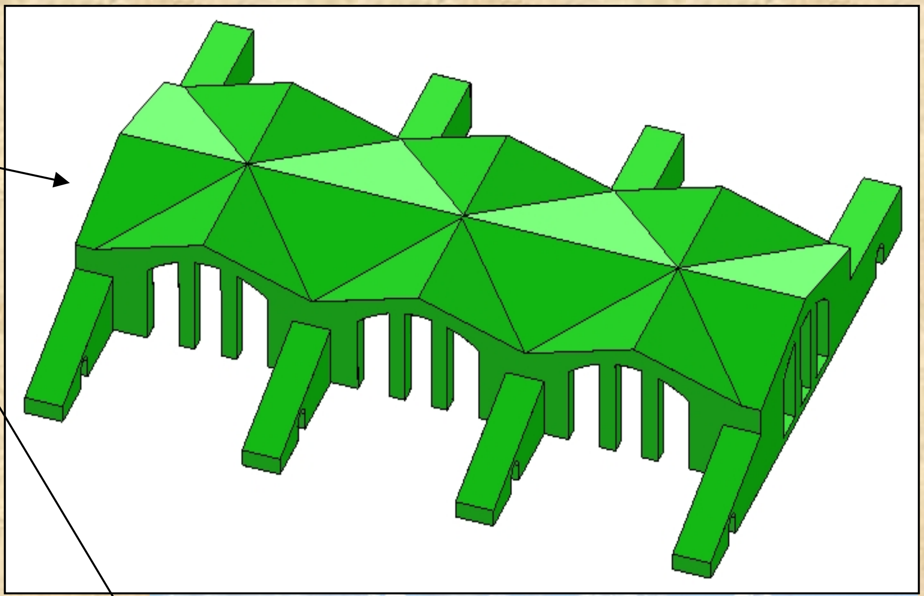
Basilica of Maxentius

Frigidarium - vault extrados

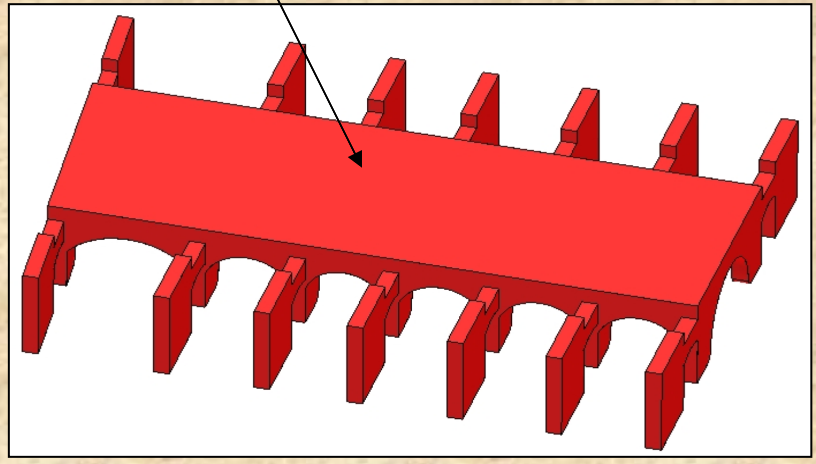


Gable extrados follows the contour of the vault intrados with substantial weight reduction

vault extrados
Frigidarium

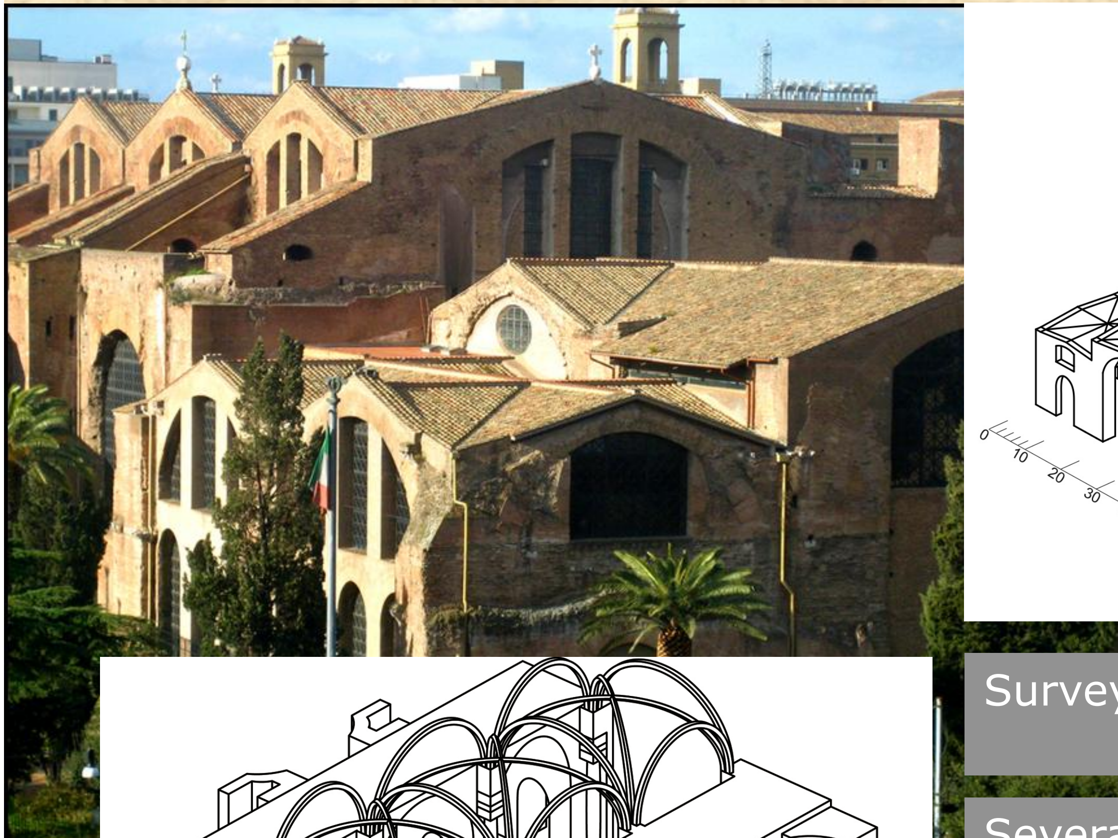


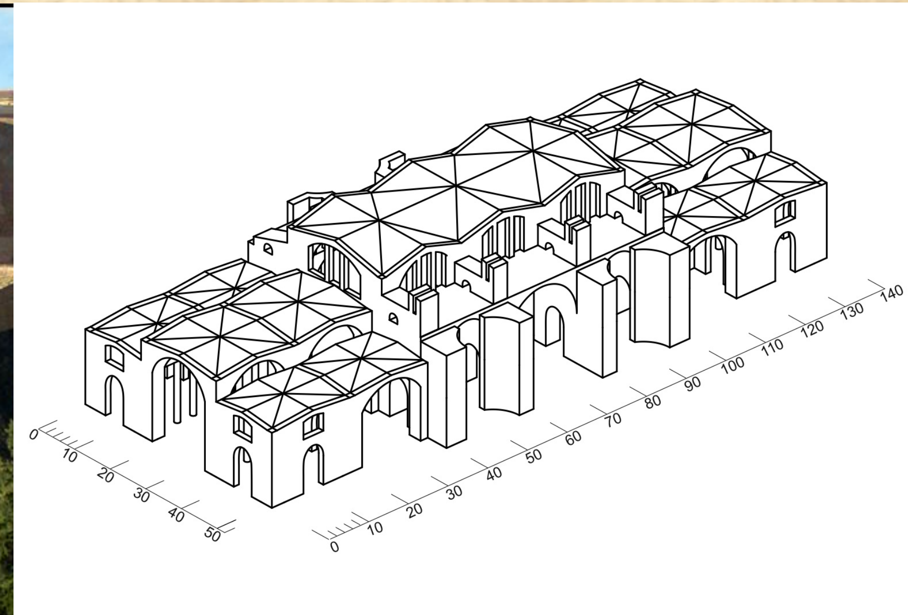
vault extrados
Great Hall

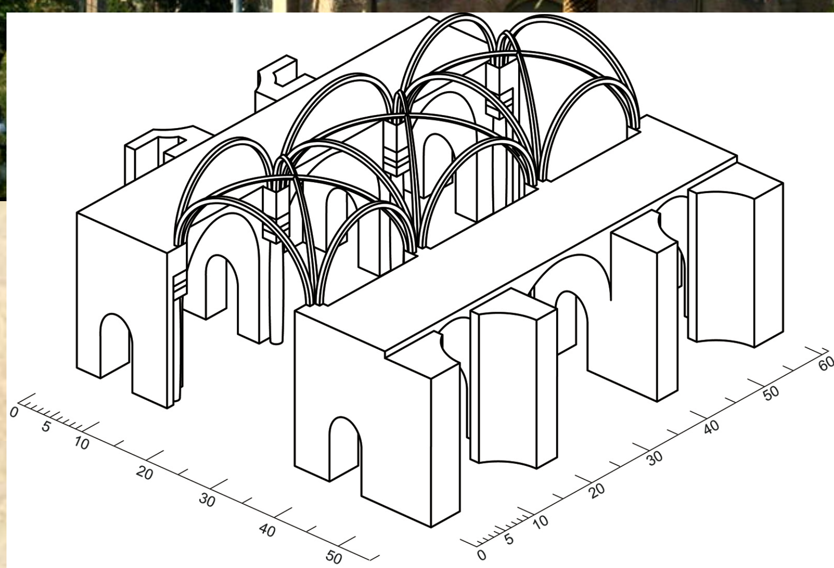


R.Perucchio

Baths of Diocletian - Frigidarium (AD 298-305)





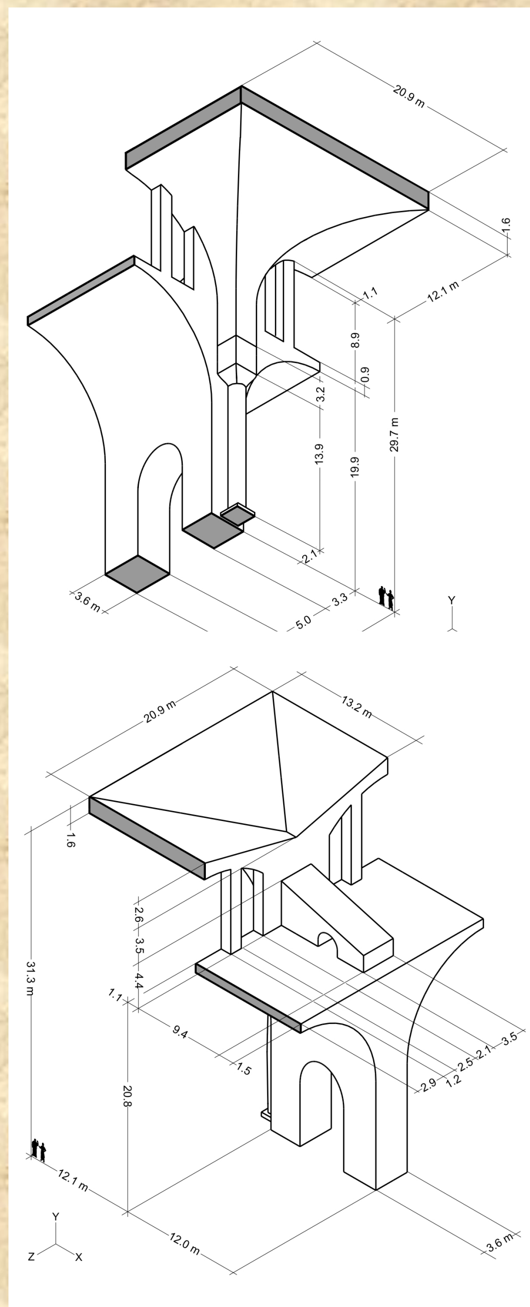


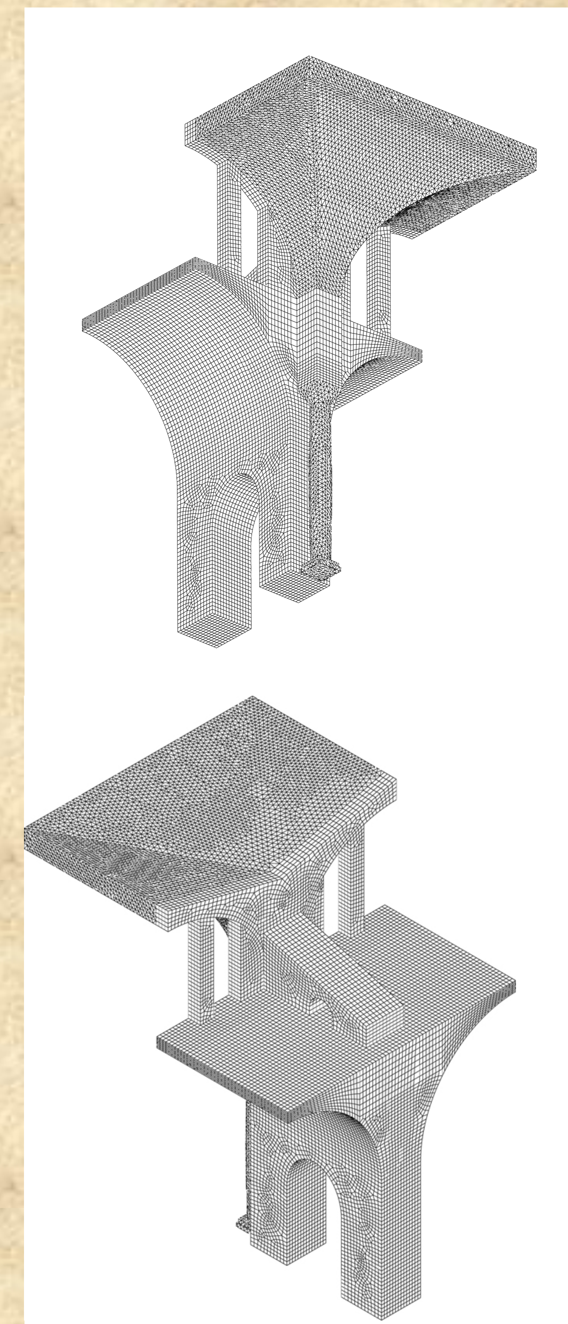
Survey 3D solid model created in RHINO

Several FEM 3D meshes (solid tetrahedral and hexahedral elements) created in ABAQUS CAE

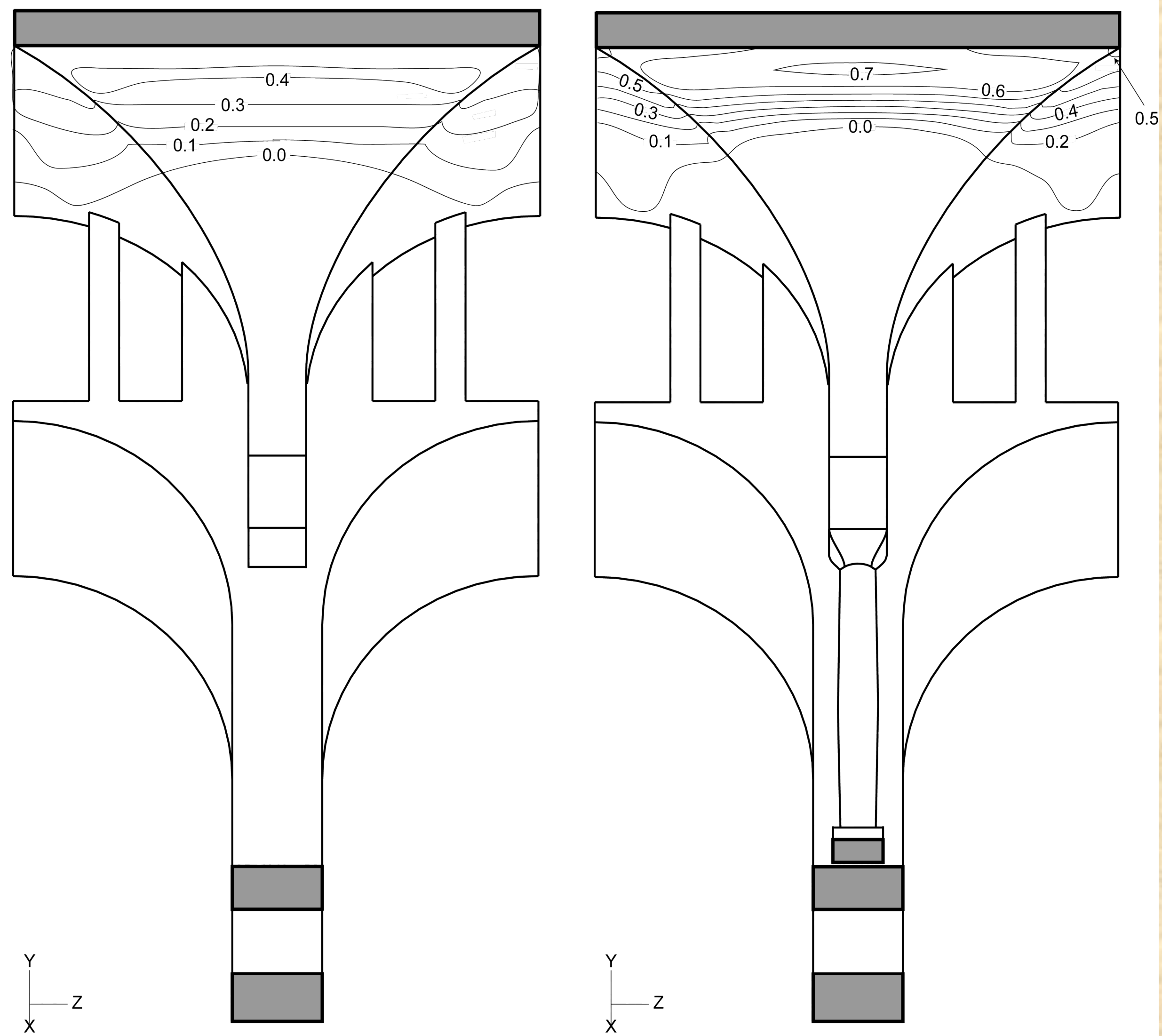
3D solid model of vault brick ribs created in ABAQUS CAE

ARCHAEOLOGY TECHNOLOGY AND HISTORICAL STRUCTURES



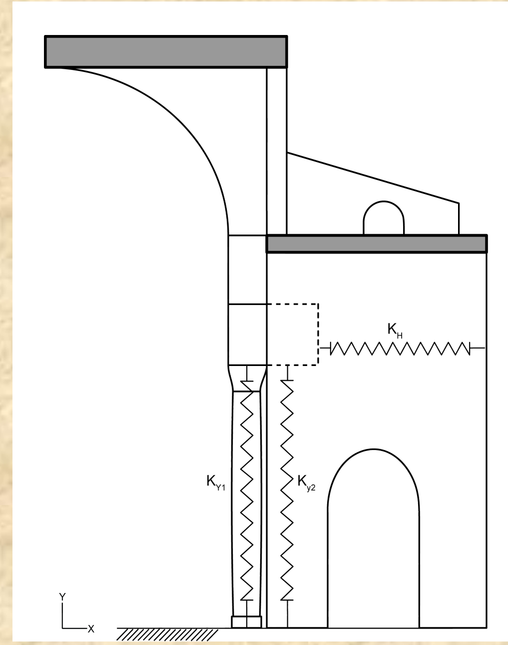


Diocletian's *Frigidarium* - vault structural system: columns



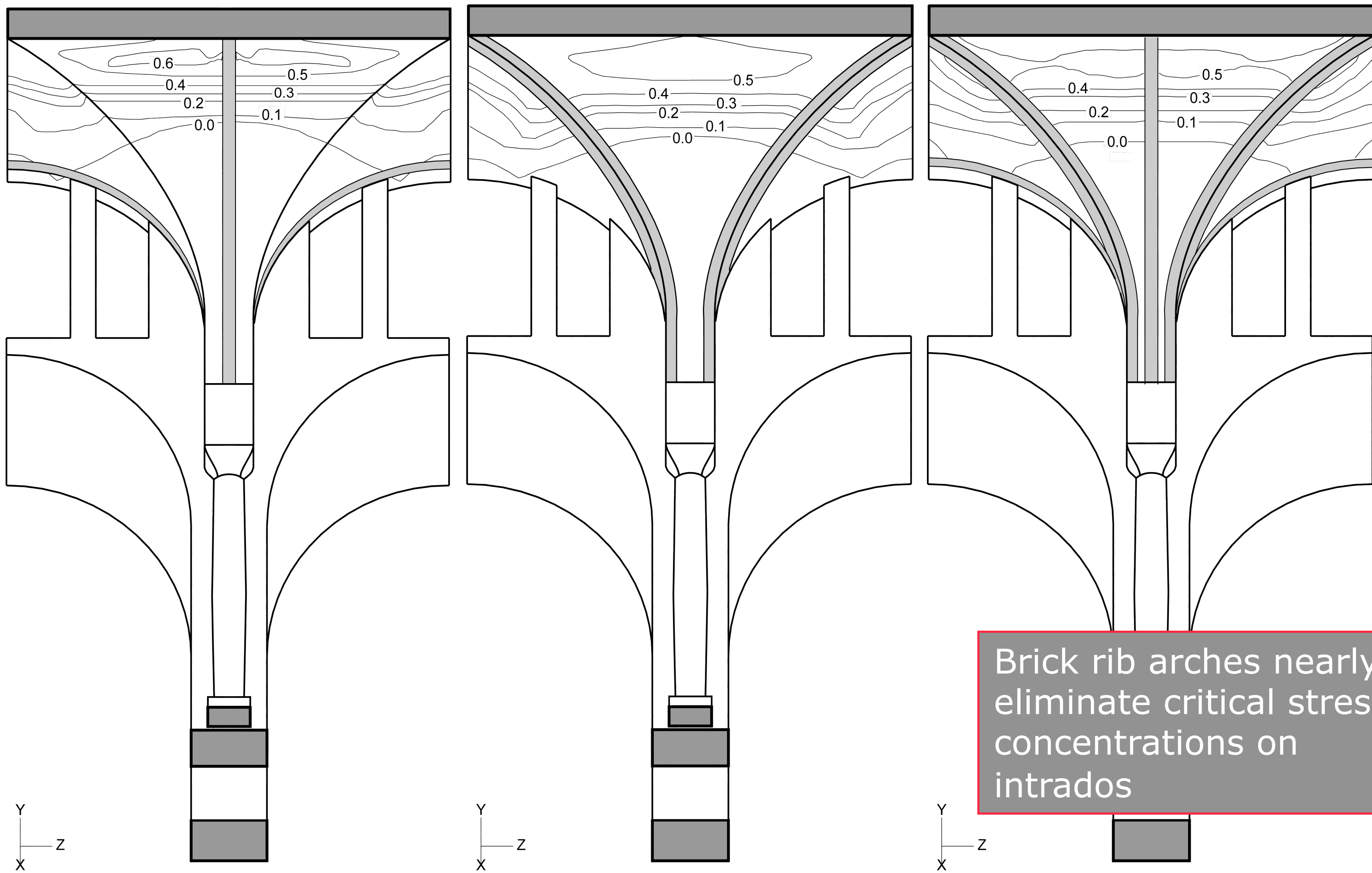
Maximum principal stresses (tensile) on vault intrados: without column (left) and with column (right)

Column stiffness produces critical tensile stresses on vault intrados



Granite considerably stiffer than *opus caementicium*

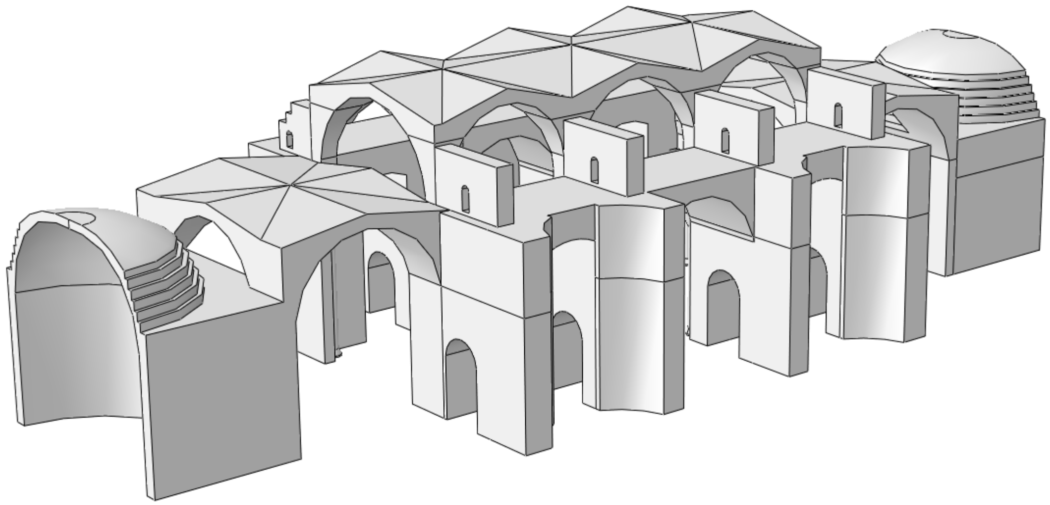
Diocletian's *Frigidarium* - vault structural system: rib arches



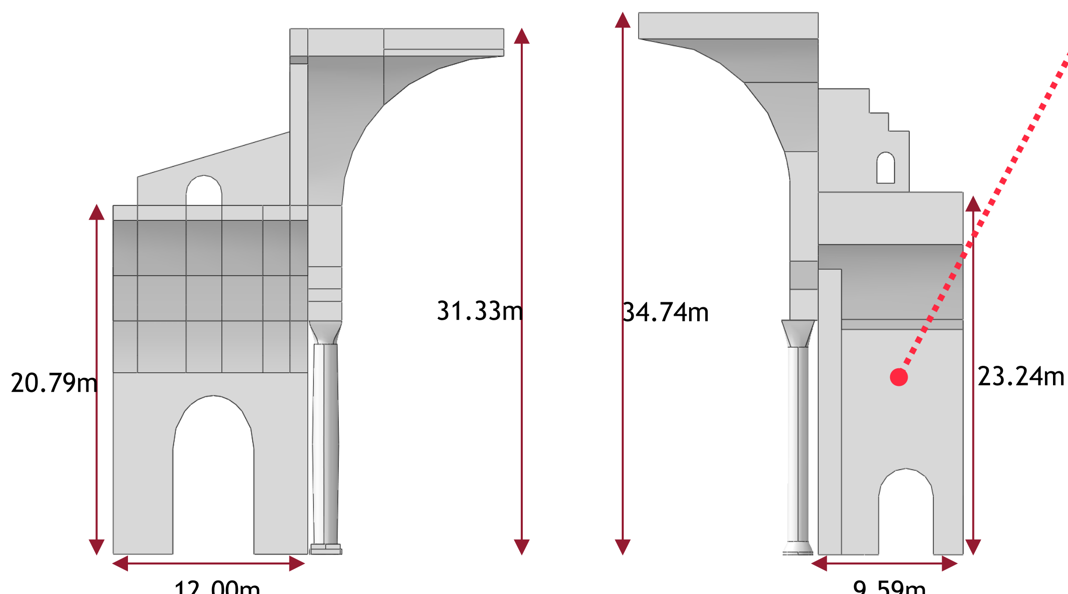
Brick rib arches nearly eliminate critical stress concentrations on intrados

Maximum principal stresses (tensile) on vault intrados with brick rib arches

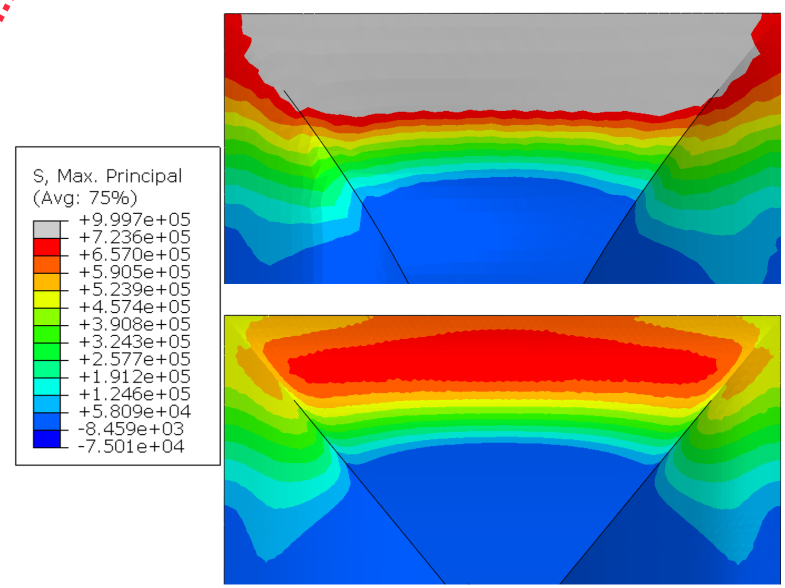
Baths of Caracalla - *Frigidarium* (AD 212-216)



Reducing shear wall depth produces high critical tensile stresses on vault intrados

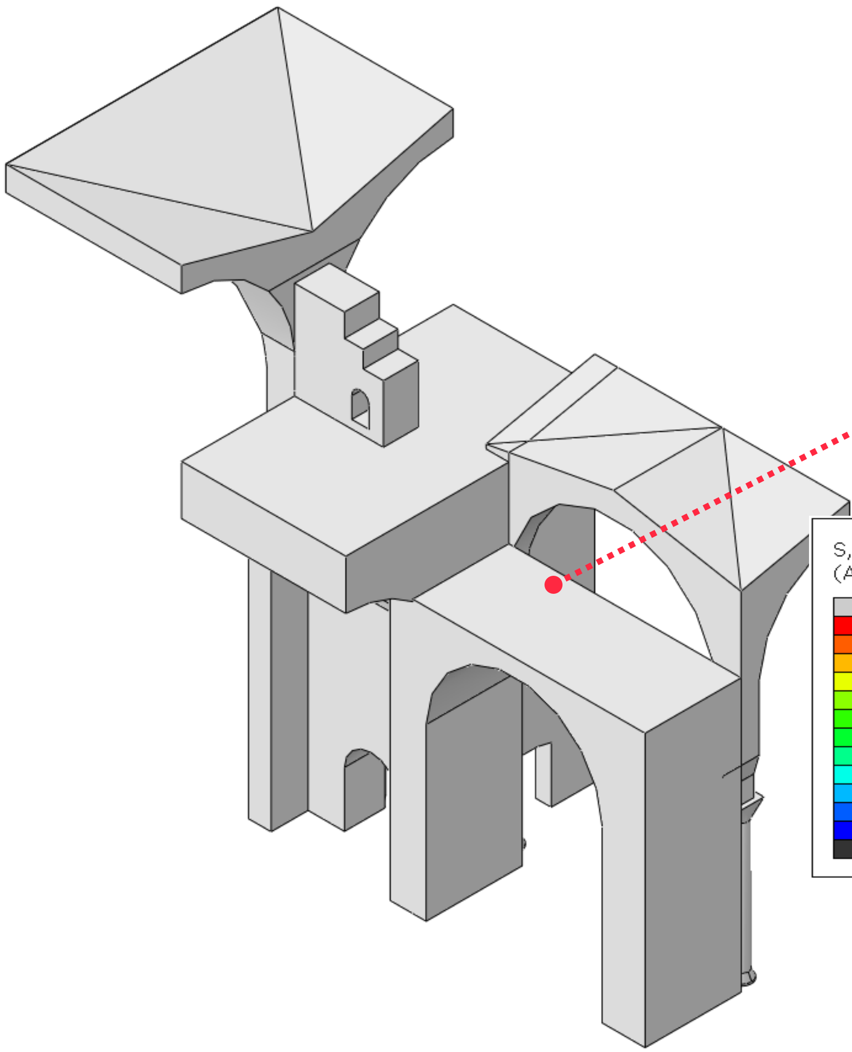


Frigidarium - modular section elevation: Diocletian's (left) and Caracalla's (right)

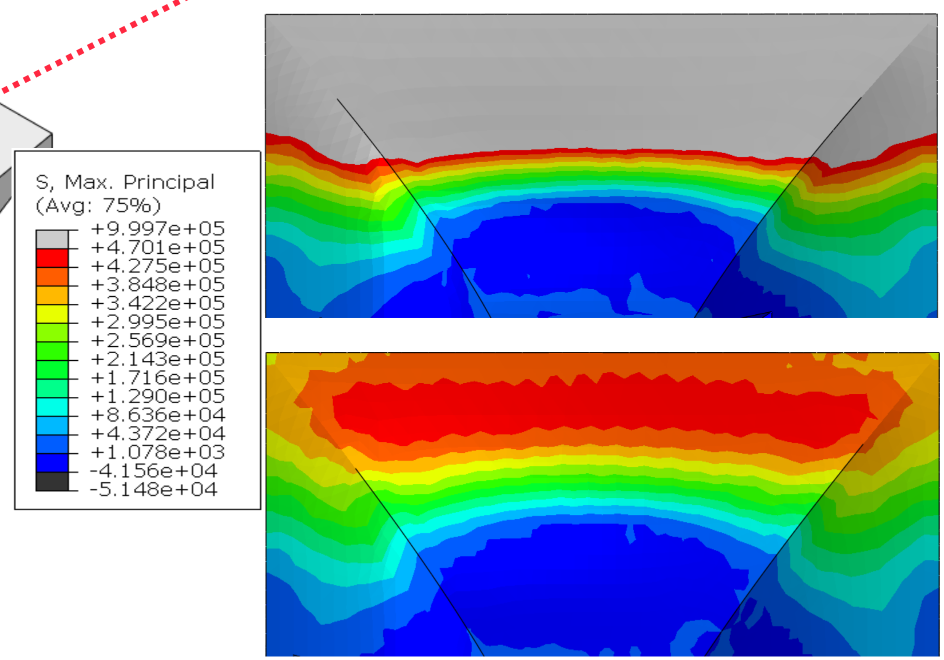


Maximum principal stresses on vault intrados: Caracalla's (top) and Diocletian's (bottom.

Baths of Caracalla - *Frigidarium* with *tepidarium* (AD 212-216)



Lateral support provided by *tepidarium* eliminates critical tensile stresses on vault intrados



Frigidarium - modular section with the *tepidarium* included

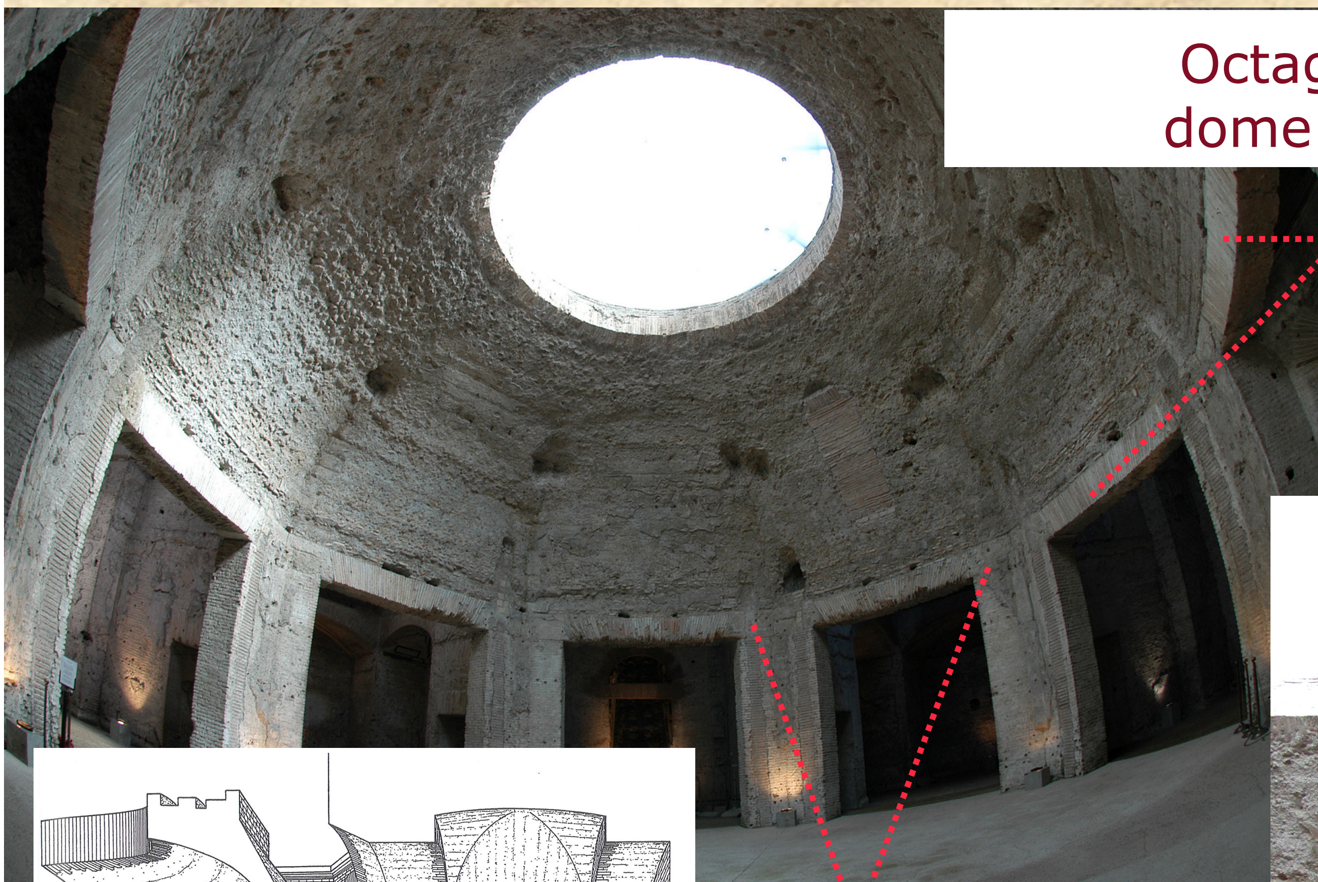
Maximum principal stresses on vault intrados: without *tepidarium* (top) and with *tepidarium* (bottom)

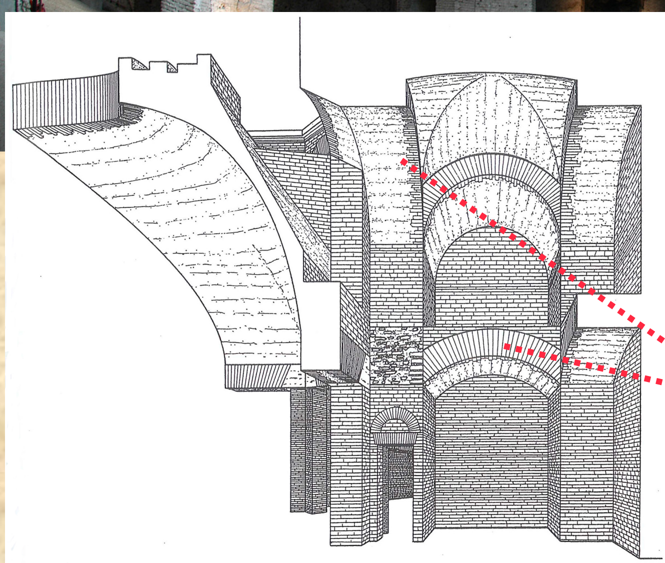
Conclusions

- The structural design of the vault of the Frigidarium evolved from the design of the Great Hall.
- The genealogy is evident in the use of contrasting walls, contrasting arches, and supporting blocks.
- The evolutionary character is shown by the repositioning of the arches, the constraining of the blocks, and the higher elevation of the contrasting walls.
- The success of this new design (structurally and functionally intact after 17 centuries) indicates the level of maturity achieved by Roman structural engineering.

Domus Aurea (64 - 68 AD) Octagonal Room

Octagonal Room - dome construction





platbands
(bricks and concrete)

timber framework

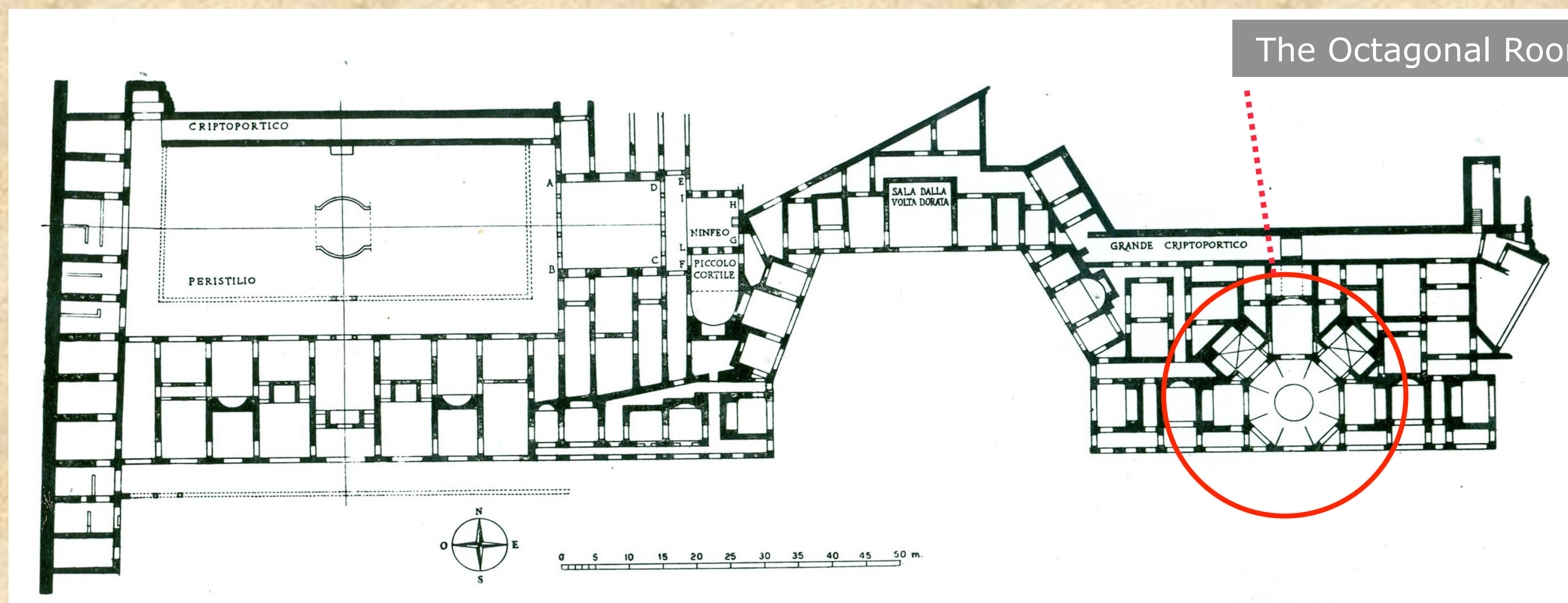
supporting blocks
(travertine)

contrasting arches
(bricks and concrete)

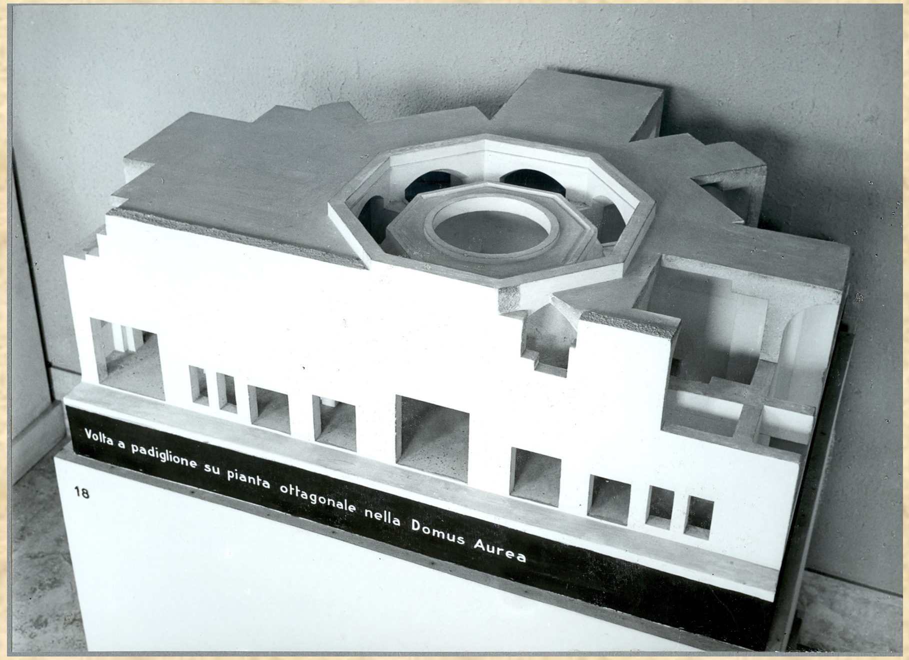
R.Perucchio

Nero's *Domus Aurea* - pavilion on the Oppian Hill

The Octagonal Room



- Entirely built with Roman pozzolanic concrete (*opus caementicium*)
- Designers: Celer and Severus ("magistri et machinatores," Tacitus, Annales, 15, 42-43)
- Abandoned after Nero's death (68 AD)
  Filled with rubble and encased in the substructure of Trajan's Baths (104 - 109 AD)
- Octagonal Room discovered in 1930's

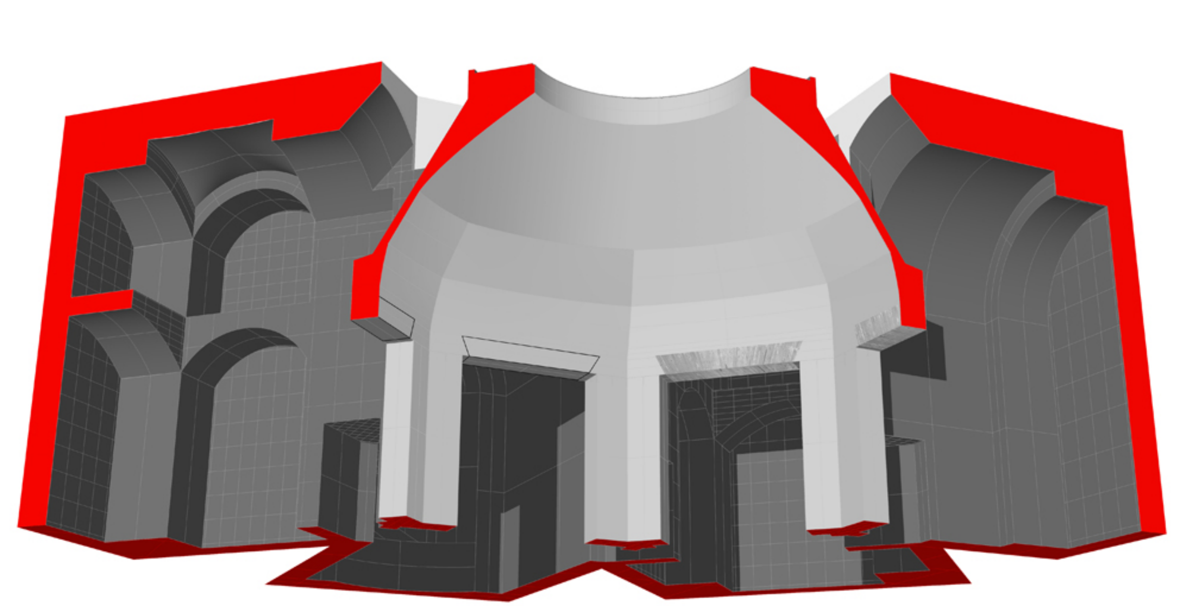


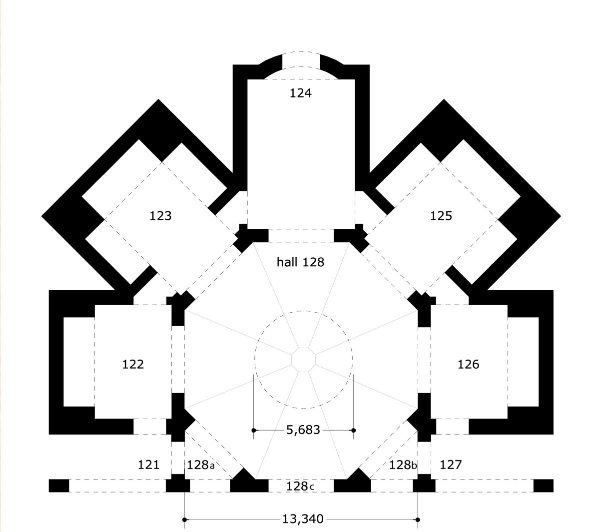
Octagonal Room - architectural layout

octagonal base:
diam. avg circumscribed circle 1442 cm
diam. avg inscribed circle 1340 cm

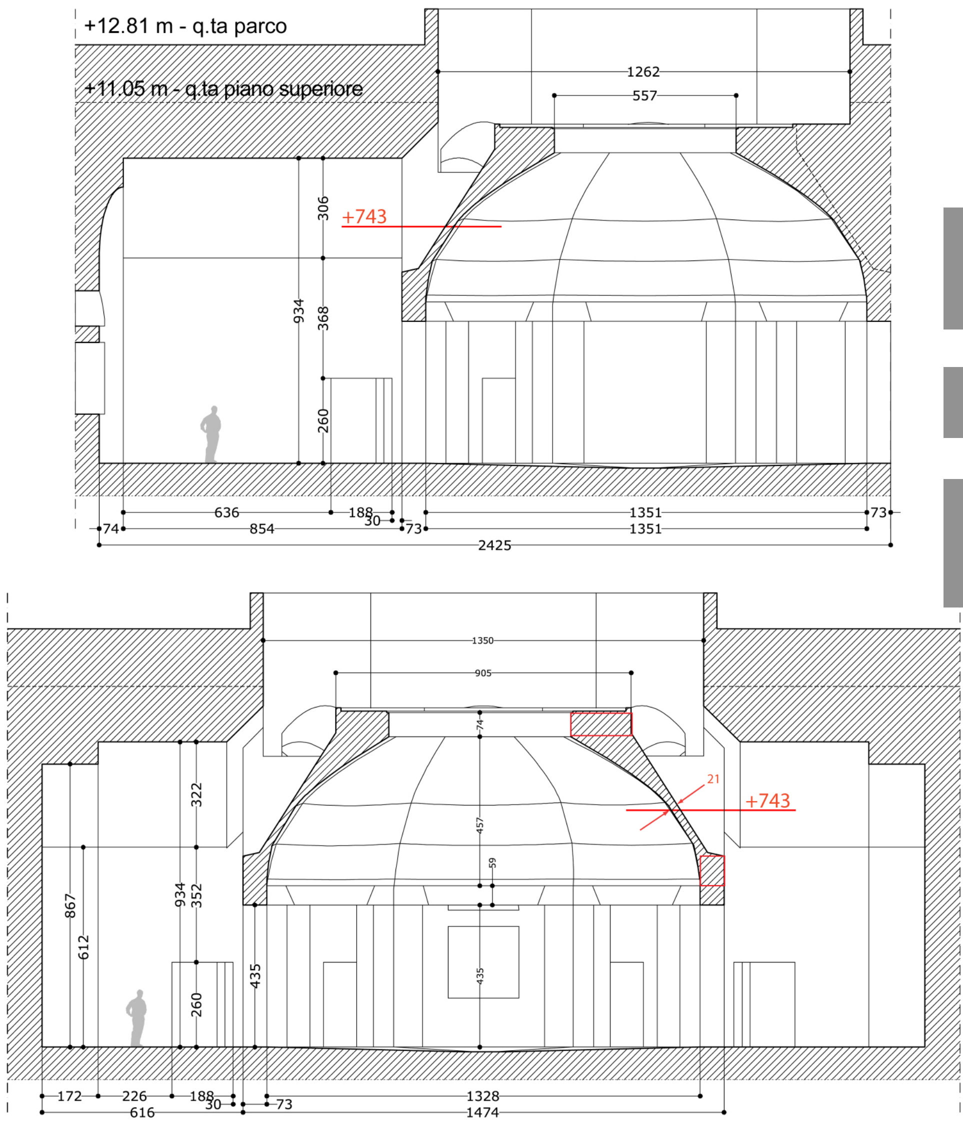
circular oculus:
diam. 557 cm

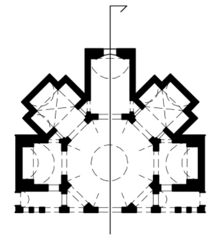
height floor to extrados of oculos
1044 cm





radially oriented rooms

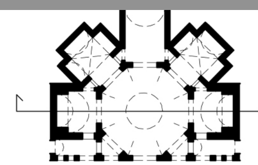


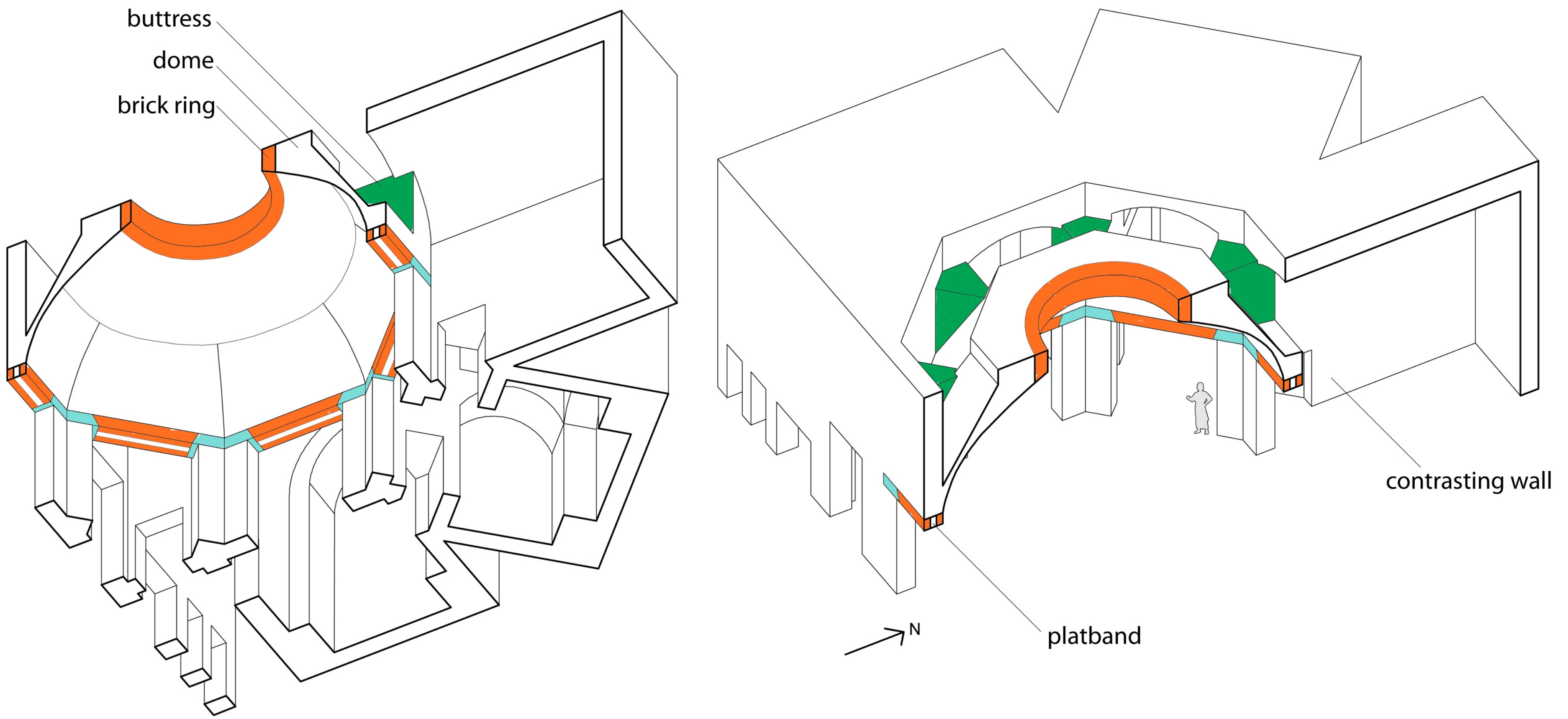


Complete new survey of intrados and extrados

Dome minimum thickness only 21 cm!!

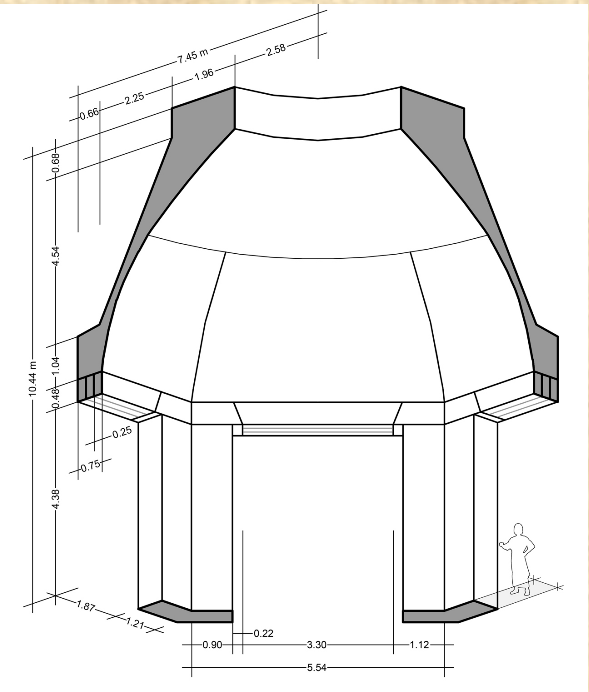
Dome in excellent structural state: no macro fractures

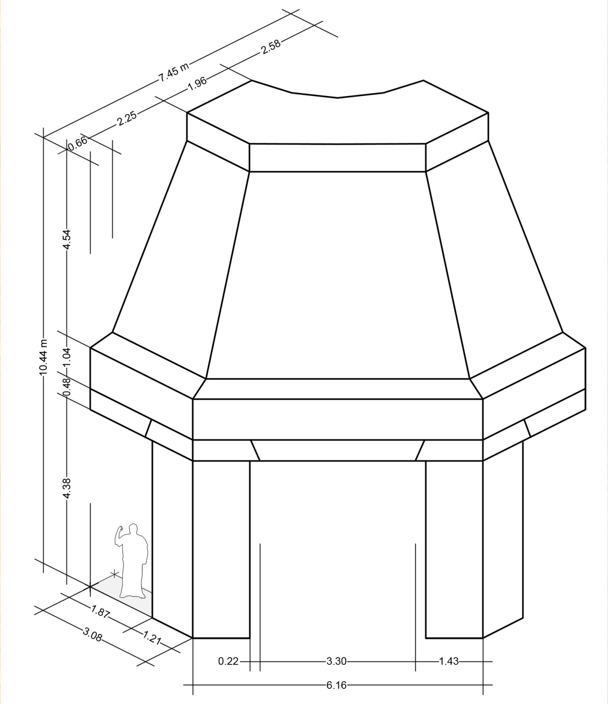


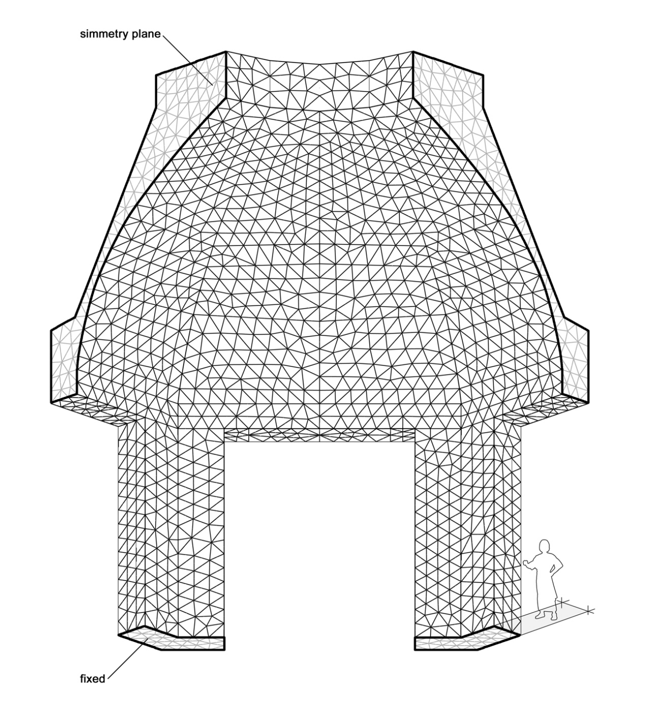


Evaluate structural function for dome static stability of (a) buttresses, (b) contrasting walls, and (c) platbands

Strategy:
analyze isolated dome, then add contrasting elements,
examine levels of principal stresses (maximum tension and compression) to determine local critical conditions

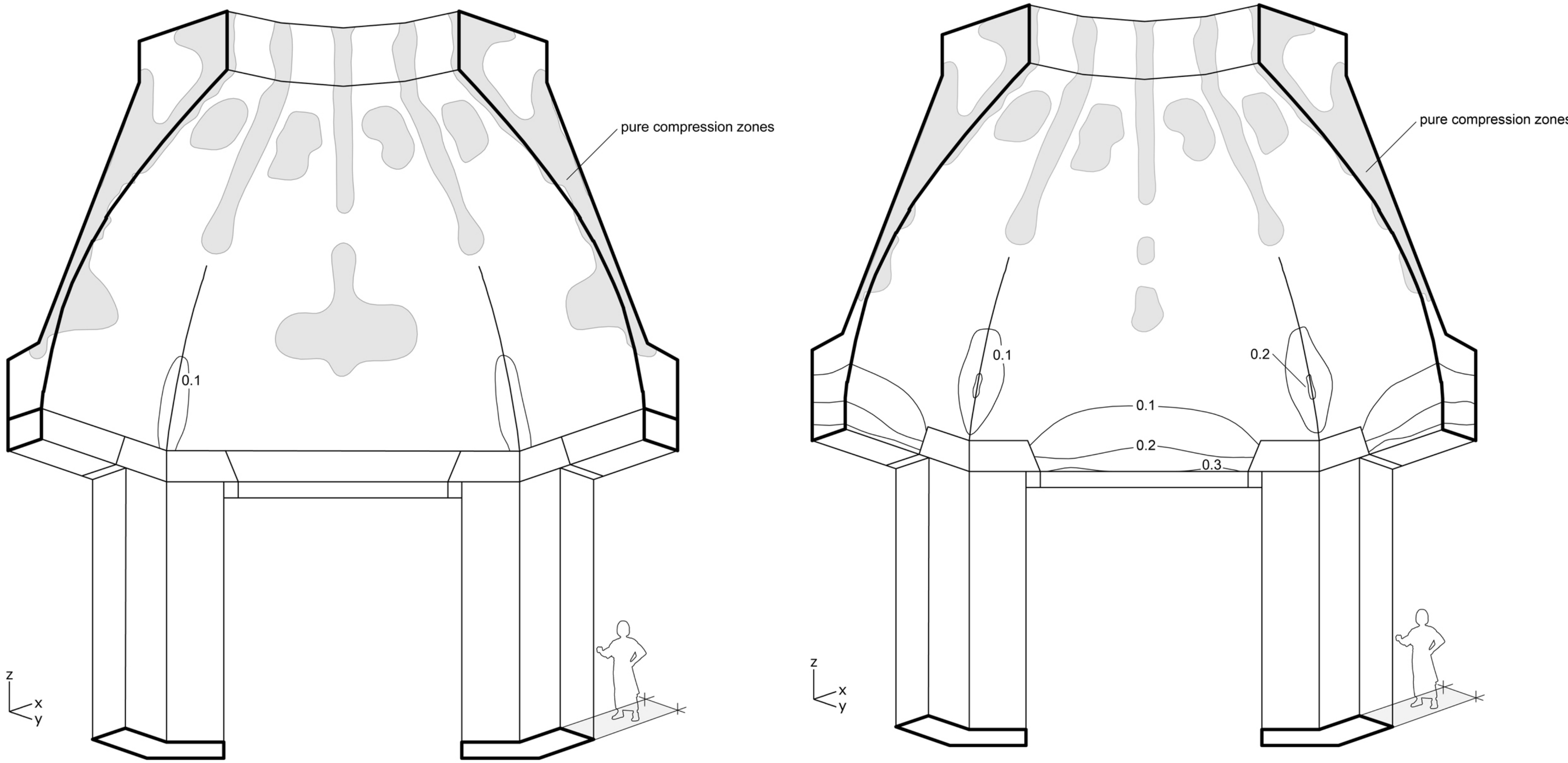




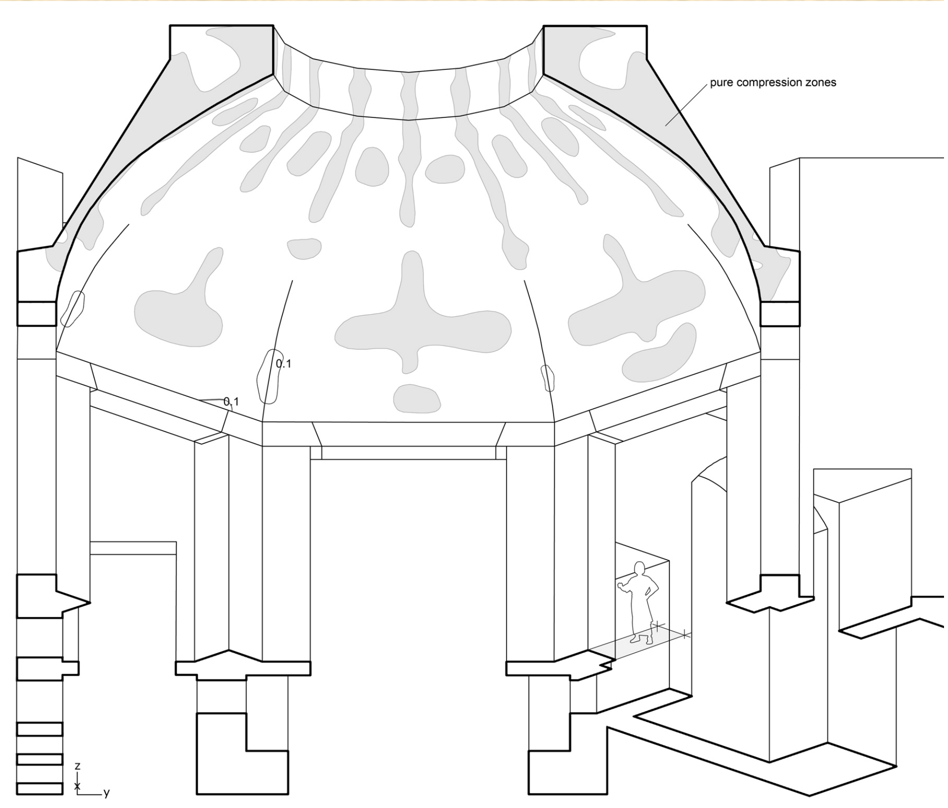


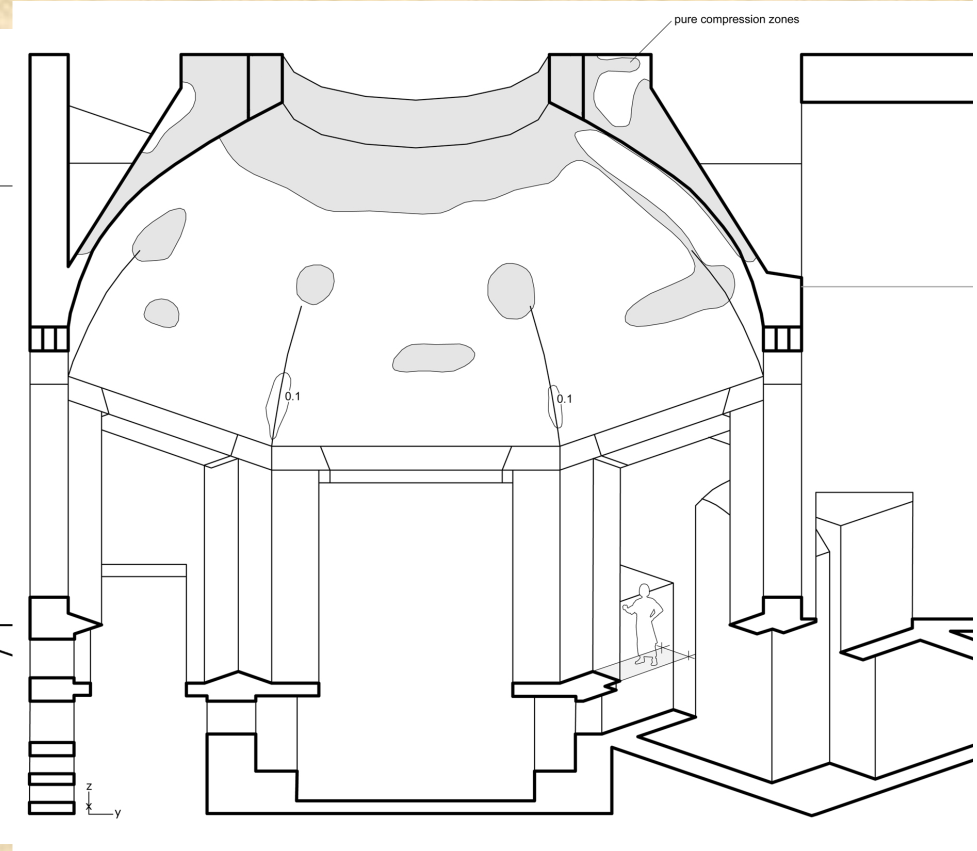
Survey 3D solid model created in RHINO

FEM 3D mesh (solid tetrahedral elements) created in ABAQUS CAE



Maximum principal stress contours: with platbands (left) and without (right)
dome primarily in compression
- isolated nuclei of tensile stresses well below fracture strength
- stiff platbands help but are not critical
- NO LOCAL DAMAGE (FRACTURE) CONDITIONS





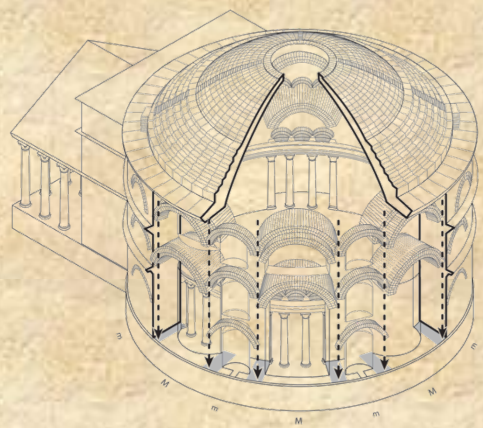
Maximum principal stress contours: with the addition of contrasting walls (left) and of buttresses on the extrados (right):

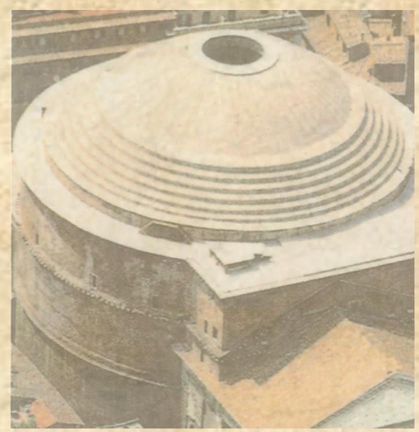
- tensile stresses only marginally reduced

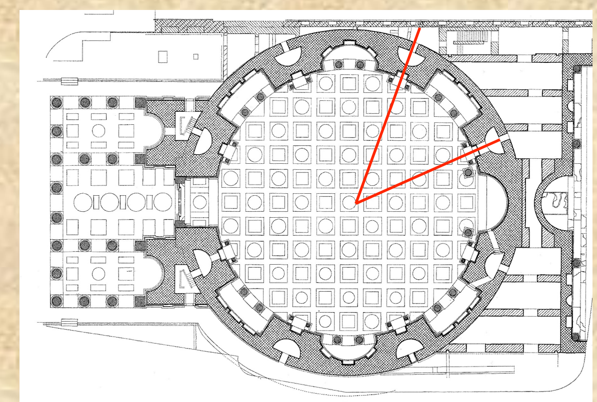
Contrasting walls and buttresses are not required for static stability.
(Dynamic analysis indicates that they are also ineffective to prevent overturning)

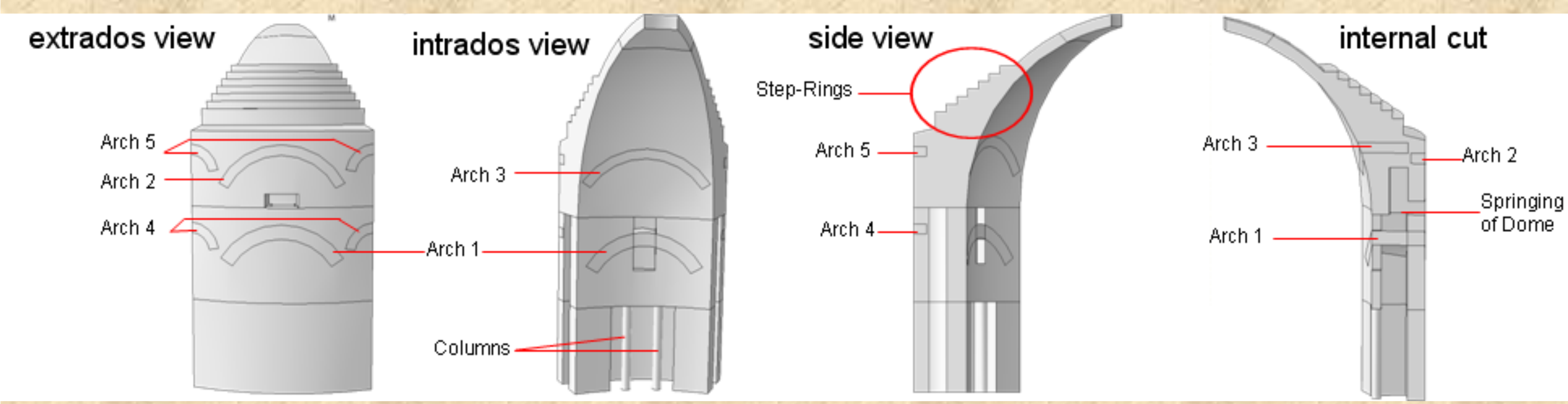
Pantheon - evaluation of relieving arches and step rings

- Model is structurally accurate (structural relieving arches are included)
- The dome and the supporting structures are modeled as pozzolanic concrete, while columns are marble, and the structural arches are either concrete or brick.









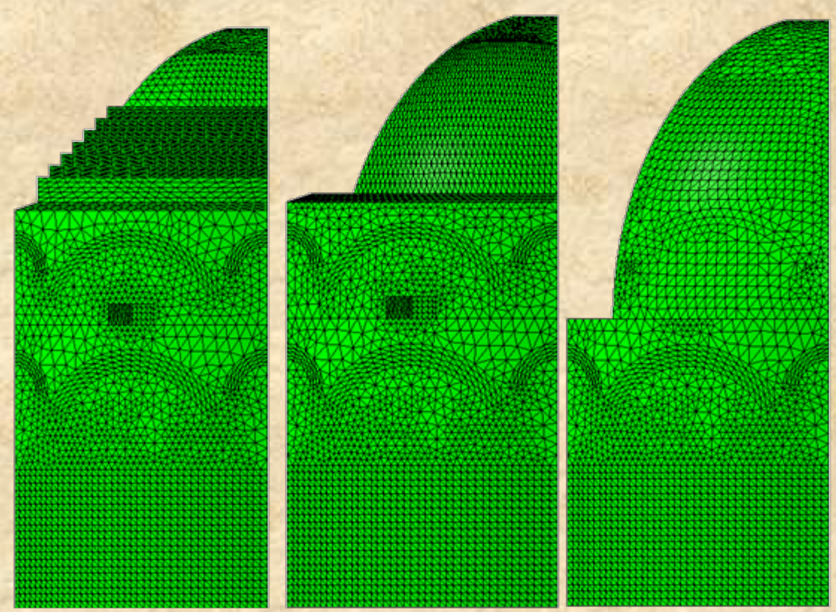
- Use 3-D FE to determine the stress fields due to static gravitational loading
- Assume linear elastic behavior throughout the structure
- Material properties: pozzolanic concrete E=3GPa, ρ = 1500 kg/m^3, v= 0.20, $\sigma_{tensile}$ =0.5MPa

Brick E=15 GPa, ρ = 2000 kg/m^3, v= 0.25

Marble E=75 GPa, ρ = 2800 kg/m3, v= 0.25

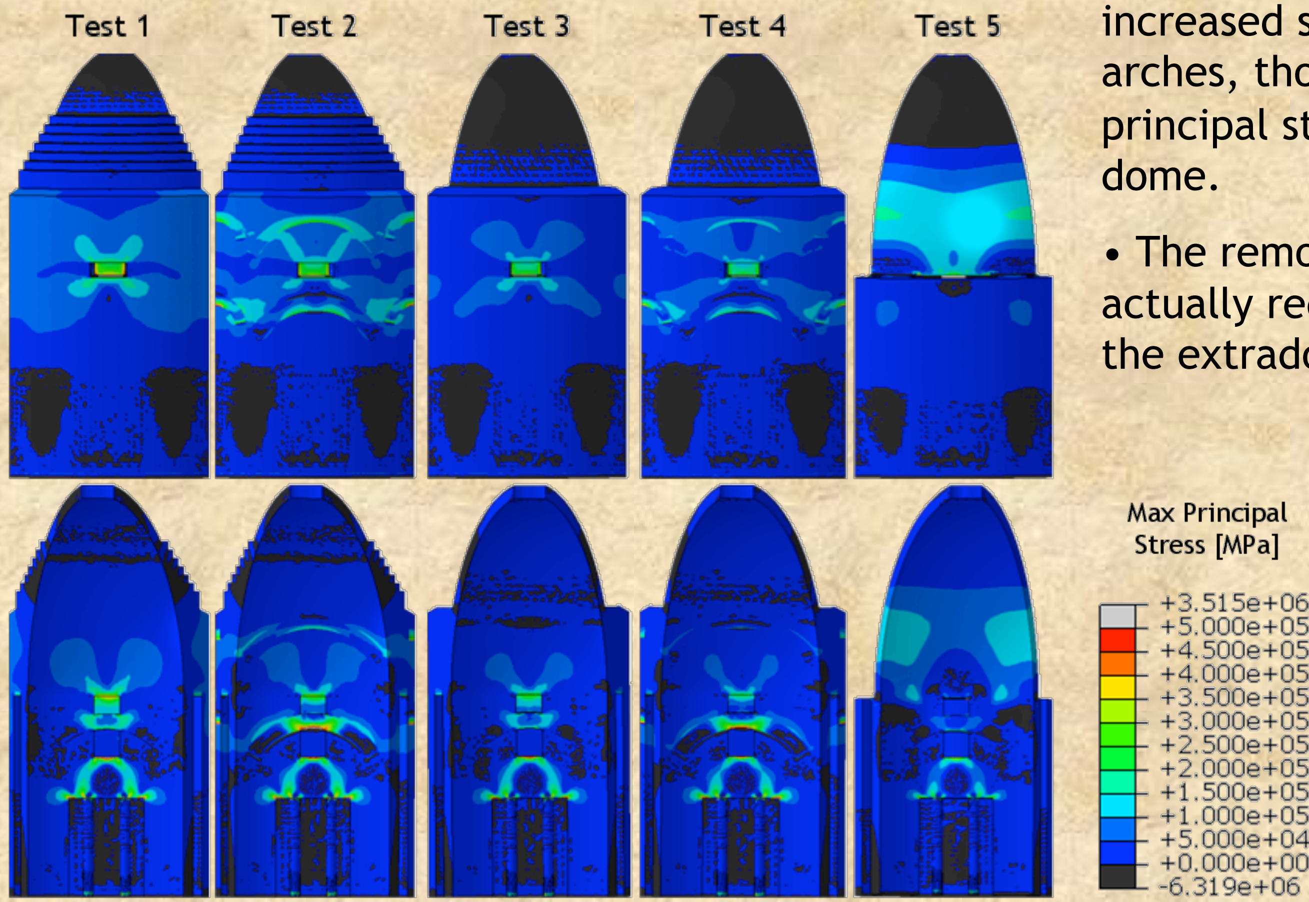
Test	Description	Computed Maximum Principal Stress in Extrados of Dome
1	•Arches modeled with concrete •Step-Rings included	0.071 MPa
2	•Arches modeled with brick •Step-Rings included	0.067 MPa
3	•Arches modeled with concrete •Step-Rings removed	0.048 MPa
4	•Arches modeled with brick •Step-Rings removed	0.041 MPa
5	•Arches modeled with concrete •Material above springing of dome on extrados is removed	0.150 MPa

Table 1- Details of FEA tests.



Left: Mesh developed for analysis of Tests 1 and 2. Center: Mesh developed for analysis of Tests 3 and 4. Right: Mesh developed for Test 5.

Maximum Principal Stresses Under Gravitational Load



Top: Maximum principal stresses on extrados. Bottom: Maximum principal stresses on intrados.

- The use of brick arches causes an increased stress concentration in the arches, though it slightly decreases the principal stresses in the extrados of the dome.
- The removal of the step-rings actually reduces the overall stresses in the extrados.
- Test 5 indicates that even with no material above the springing of the dome, the stresses in the extrados remain well within the tensile strength of pozzolanic concrete.
- Compressive stresses are noncritical to the structure.

Future directions in concrete vaulted structures

Mechanical testing of *opus caementicium* (re-created and actual) to measure cohesive fracture properties

Numerical (FEM) energy-based criteria for simulating static and dynamic structural collapse conditions (local and global)

Experimental techniques for measuring structural dynamic response

Numerical (FEM) nonlinear simulation of response to earthquake accelerations